THE DURATION OF TWILIGHT

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BEING advised by a lawyer friend that the only information that he could find on the duration of twilight was contained in the encyclopedia (the only information given there being to the effect that it lasted until the sun was from 18 to 19 degs. below the horizon, and that "near the equator there is very little twilight"), the writer became interested in the question but could find very little written matter, although there is no doubt that there has been considerable written on the subject and anything that the writer may say here has, perhaps, been written before many times.

The subject is of very little economic value, so there is no justification for a long discussion, but one of the economic values that could be placed on the results, is that in high latitudes the laws re carrying lights on vehicles and railway trains between sunset and sunrise might be considerably modified for at least four months during the year.

If the statement in the encyclopedia is correct, that twilight lasts until the sun is 18 degs. or more below the horizon, then at the equator at the time of equinox, there would be 1 hr. 12 mins. or more of twilight. From various personal observations the writer is of the opinion that these figures are too large.

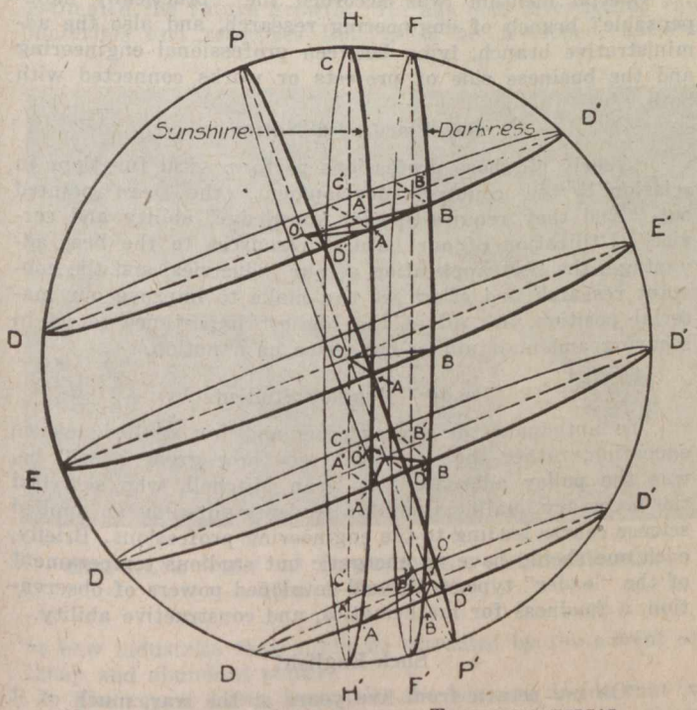

FIG. 1—DIAGRAM ILLUSTRATING TRIGONOMETRICAL SOLUTION FOR THE DURATION OF TWILIGHT

There being no absolute darkness, it is, of course, a very difficult matter to say when twilight ends and darkness begins, or vice-versa. Assuming that we are justified in believing that twilight has ended when no difference can be noticed in the brightness of the western horizon from any other portion of the horizon, and when the usual number of stars are visible (when there is no moonlight), then in the writer's opinion it would be nearer the facts to say that twilight ends when the sun is 15 degs. below the horizon.

To enable people who may be interested to judge of this matter for themselves, the writer has prepared two diagrams giving the duration of twilight at various latitudes and seasons of the year,—one on the assumption that the angle of twilight is 18 degs. and the other on the assumption that the angle of twilight is 15 degs.

For the benefit of those who may not care to take the trouble of working this problem out for themselves, the writer herewith gives his solution. The writer was of the opinion that it would be a problem in spherical trigonometry, but after a little study it was seen that it could be solved by plain trigonometry.

Let Fig. 1 represent the earth; O, the centre of same; EE', plane of the equator; HH', a plane perpendicular to the rays of the sun passing through the centre of the earth. Then, neglecting refraction and inequalities of the earth's surface, where this plane cuts the surface of the earth, is the loci of points of sunset or sunrise.

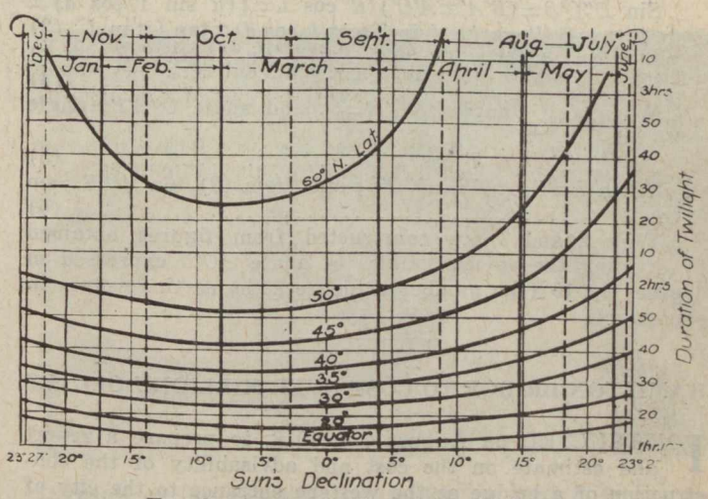

FIG. 2—ANGLE OF TWILIGHT, 18 DEGS.

Diagram indicating the duration of twilight at various seasons of the year and at various latitudes in the northern hemisphere, assuming that the angle of twilight is 18 degs. (that is, 1 hr. 12 mins. at the equator when the declination of the sun is zero—about March 22nd and September 22nd).

FF' is a plane parallel to plane HH' and marking the dividing line between twilight and darkness.

PP' is a plane through the poles of the earth and passing through the common axis of planes EE' and HH'.

DD' is a plane of any parallel of latitude.

Let R = radius of the earth.
L = angle of latitude of plane DD'.
d = angle of declination of pole PP'.
T = angle of twilight (given by some authorities as 18 to 19 degs.).

The problem is: Having given angle of twilight (T), to find the duration of twilight at various latitudes at various seasons of the year; that is to find the value of angle $AO'B$.

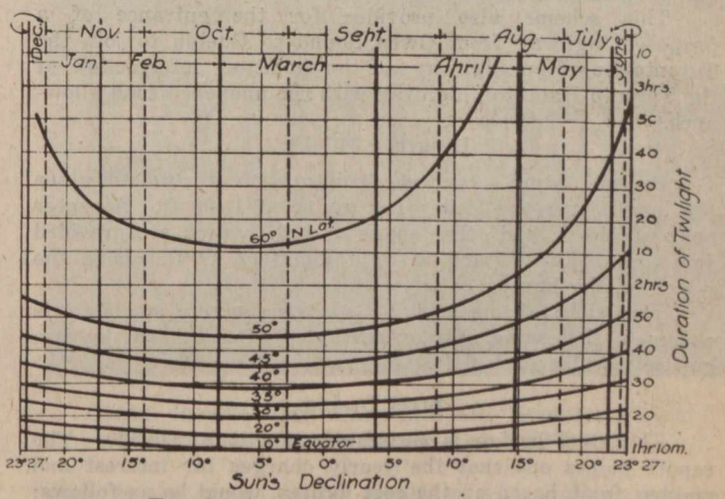

FIG. 3—ANGLE OF TWILIGHT, 15 DEGS.

Diagram indicating the duration of twilight at various seasons of the year and at various latitudes in the northern hemisphere, assuming that the angle of twilight is 15 degs. (that is, 1 hr. at the equator when the declination of the sun is zero—about March 22nd and September 22nd).

The perpendicular distance between plane FF' and HH' is $FC = B'C' = R \sin T$. The constant distance for any given value of d between plane FF' and HH' measured in plane DD' is

$$B'A' = B'C' / \cos d = R \sin T / \cos d \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$