$$\therefore S-\frac{1}{3}=\frac{1}{8}+\frac{1}{13}+\frac{1}{18}+\ldots\ldots+\frac{1}{5n+3}.$$

$$\therefore \frac{1}{3}=5\left\{\frac{1}{3\cdot 8}+\frac{1}{8\cdot 13}+\ldots\ldots+\frac{1}{(5n-2)(5n+3)}\right\}+\frac{1}{5n+3}$$

$$= 5 \text{ times sum of series} + \frac{1}{5n+3}.$$

$$\text{Sum of series} = \frac{1}{15}-\frac{1}{5(5n+3)}.$$

Sum ad. inf. $=\frac{1}{15}$, second fraction disappearing when n becomes infinite.

11. Putting $a=0$, the determinant becomes

$$\begin{vmatrix} bc, & 0, & 0 \\ b^2-c^2, & 0, & 0 \\ c^2, & c^2, & b^2 \end{vmatrix}$$

which is evidently zero, each minor determinant vanishing. Therefore a is a factor.

Similarly, b and c are factors. Again putting $a+b$ for $-c$, the determinant becomes

$$\begin{vmatrix} -b(a+b), & a(a+b), & -ab \\ -a^2-2ab, & -a^2-2ab, & -a^2-2ab \\ c^2, & c^2, & c^2 \end{vmatrix}$$

which is evidently also zero, since the minor determinants

$$\begin{vmatrix} -a^2-2ab, & -a^2-2ab \\ c^2, & c^2 \end{vmatrix} \text{ all vanish.}$$

For information on Determinants see Loudon's and Gross's Algebras.

Communications received are held over until next month.

Practical Department.

ARITHMETIC—ITS STUDY MADE EASY AND PLEASANT.

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What I have to advance on this subject will be condensed within the narrowest limits possible; and, as far as consistent, I prefer to let eminent scholars speak for me.

I. The subject of arithmetic itself is easily comprehended. We shall realize this if we remember—

(1.) That *unschooled* men in all ages have understood and performed its *vital* processes.

(2.) That there are excellent accountants among us who have never learned a rule in arithmetic; nor have they ever studied any but a child's arithmetic.

(3.) That many of our best business men have never advanced in written arithmetic beyond fractions.

II. Its fundamental principles are few and simple.

(1.) "Arithmetic is founded on Notation."—Ray's Higher Arithmetic. "When we enter into the spirit of the methods of arithmetic we perceive that they all flow clearly and simply from the very principles of Numeration and a few axioms."—Le Verrier, the great French mathematician. "Every change we make upon the *value* of a number must *increase* or *diminish* it."—Thomson's New Practical.

III. Its fundamental operations are but two; Addition and Subtraction. See Duncan's, Jamieson's and Wilson's Logic.

(1.) "Multiplication is a short method of *adding* equal numbers."—See Quackenbos's, Davis's, Thomson's and Felter's Arithmetics, Day's Recent Logic, and Rev. J. Currie, of Edinburgh Training College.

(2.) "Division is but a different kind of subtraction."—See Wickersham's Methods, DeGraff's School Room Guide, Sheldon's Elementary Instruction and Ray's Arithmetic.

(3.) "Multiplication, Division, Involution, Evolution, etc., are only more useful because shorter methods to the same results."—Prof. Wilson, Cornell University Logic.

If things are so (and who will dispute them), why is it that pupils in all our schools spend so large a portion of their school days in the study of arithmetic? Here is one answer:

"We do not hesitate to acknowledge that the teaching of elementary mathematics has lost its former simplicity, and assumed a complicated and pretentious form which posseases no advantages and is full of inconveniences."—Professor Gillespie, Union College.

"In New England, the science of arithmetic is taught backward, beginning with reasoning instead of observation; and it is hampered with factitious difficulties, produced by a variety of *unessential* names and processes."—Ex-President Hill, Harvard University.

To restore arithmetic to its former simplicity, and to render its study easy and agreeable—

(1.) Omit from our books on this subject, or remand them to an Appendix, most or all of the following titles, which are little used in the *actual* business of life:—The English mode of numeration; most contracted methods; greatest common divisor; true remainder; different scales of notation; proof by casting out the nines and the elevens; continued fractions; periodical or circulating decimals; compound and conjoined proportion; compound interest; annuities; modes of computing interest in other states and nations; life and marine insurance; general average; stock jobbing; arbitration of exchange; alligation; permutations and combination; duodecimals; methods of analysis by position, and all those parts which treat merely of curious properties of numbers. But, care should be taken to "retain and increase those parts which furnish commercial expedients, or are essential to a thorough preparation for the actual business of life." Says Prof. DeGraff: "As the majority of pupils leave school at the average age of twelve years, they should be drilled on the subjects which they will be obliged to use through life. They should be taught to solve problems they will meet in *real* life."

(2.) "Apply the formulas of mental arithmetic to the solution of questions in written arithmetic.—Felter's Arithmetic. "There should be no difference between the analysis of a problem in mental and written arithmetic."—DeGraff.

(3.) All mere *rote* teaching and learning should, at once, be abandoned. Long ago said Montaigne: "To know by rote is no knowledge."

(4.) Mere *rule* teaching should also be abandoned. Said Locke, two hundred years ago: "Nobody has made anything by hearing of rules, or laying them up in his memory. *Practice* must settle the habit of doing, without reflecting on the rule." Said Diesterweg, the great German educator: "In arithmetic, prescribed rules and formulas are to be entirely annihilated. No operation not understood in its reason should be performed or learned." Warren Colburn taught how many problems may be solved without having "learnt the rules." Said Horace Mann, who visited the German schools some years ago: "It struck me that the main difference between their mode of teaching arithmetic and ours consists in their beginning earlier, continuing practice in the elements much longer, and in requiring a more thorough analysis of all questions. *There were no abstract rules or unintelligible forms of words given out to be committed to memory."—School Bulletin.*