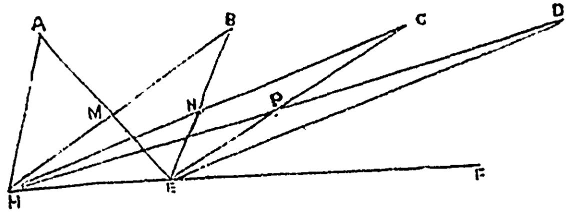
numbers of the *Journal* at hand, the substance of my refutation of Legendre is given in an Appendix to the present paper.

Proposition I.

The sum of the angles of a triangle AHE (Fig. 1) is not greater than two right angles.

FIG 1



For, produce HE to F. Bisect AE in M. Draw HMB, making MB = HM; and join BE. In like manner construct the triangle CHE; N being the middle point of BE; and CN being equal to HN. In like manner construct the triangle DHE; P being the middle point of CE; and DP being equal to PH. And so on indefinitely. Denote by S, S_1, S_2, &c., the sum of the angles of the triangles AHE, BHE, CHE, &c., respectively; and by A_1, A_2, A_3, &c., the angles HBE, HCE, HDE, &c., respectively. Then it is plain that the quantities S, S_1, S_2, &c., are all equal to one another. Also, as the number n becomes indefinitely great, the angle A_n becomes indefinitely small. For, the sum of all the angles in the series, A, A_1, A_2, &c., is less than AEF; and, since the series, A, A_1, &c., may be made to contain an indefinite number of terms, those terms which are ultimately obtained must be indefinitely small, in order that AEF may be a finite angle. But, the exterior angle DEF being greater than the interior and opposite angle DHE, S_3 cannot exceed two right angles by D. And $S_3 = S$. Therefore S cannot exceed two right angles by D or A_3. In like manner it may be proved that S cannot exceed two right angles by A_n, whatever n be. And A_n is ultimately less than any assignable angle. Therefore S cannot exceed two right angles by any finite angle whatsoever.

Cor. 1.—If a line AE (Fig. 2) be drawn from A, an angle of a triangle ADF, to a point in the opposite side; and if the sum of the