by the phrase "empirically discovered rules"? Suppose we have a rectangular surface before us—a room, a field, a figure on the blackboard—and I wish to know the magnitude of its surface.

There are but two ways of procedure—for our present purpose—and these differ *in toto.* I propose to consider one of them.

It is clear we must have a certain surface (called a unit) with whose magnitude we are familiar—itself also rectangular. I now take this unit and find, by actual trial, how many times I can lay it down on the given rectangular surface, each time in a quite new position, before I have used up all the space included within the boundary. Then, neglecting certain obvious considerations foreign to the purposes of the illustration, if it appears that the original surface does not contain the measuring unit an exact number of times, I may either neglect the piece over as inconsiderate, or I may select another and smaller unit with which to again make a similar series of measurements. Thus, by repeated use of smaller and smaller units, I at length arrive at one whose magnitude is so small that I cannot well make use of a smaller. There now appears to me to be no piece at all neglected. I call the measurement exact. But is it so? Certainly not; it is now correct to say, not that I have measured exactly, but that I have reached the limit of my measuring powers. The exactness is only relative, for I have merely to employ an individual with keener eyesight and more delicately manipulative capacity to obtain what *he* would doubtless, in his turn, call an exact measurement; and yet, though certainly more exact than mine, it is still clearly only a relative exactness. A little reflection, indeed, will convince one that there is no end to such an inquiry; no surface, concrete and actual, admits of absolutely exact measurement. Why not? Because, amongst other equally important reasons, we cannot define, with absolute precision, what we mean even by the *boundary* of such a surface. The very attempt lands us in a discussion of the subtlest problems of philosophy. Every succeeding generation of scientists, with deeper knowledge and better instruments, would improve on the measurement of its predecessors. From this aspect civilization appears as a function of the place of the decimal point. There is no finality.

Such measurements, then, as above described let us call experimental or empirical. Now observe that the measurement obtained with so much trouble applies only to this particular rectangular surface; *it gives no information about other rectangular surfaces.* Further, let us suppose that repeated measurements, by this very obvious method, of all sorts of rectangular areas, have been thus experimentally made, and the results tabulated. In addition, let the measurements of the *sides* of these rectangles be obtained in similar direct manner (by use of units of length)—whatever may be the purpose of such—and let these results chance to be tabulated alongside the others. [We presume total ignorance of geometrical *science* on the part of our practical geometricians.] Finally, let us imagine some observant individual amongst them discovering, either by chance or with intentional quest, that, if he multiplies together the numbers giving the measures of the sides, he obtains, in all the cases observed, numbers very close to those measuring the areas. [It is, perhaps, interesting to observe that the discovery of such relations would appear to be almost impossible for races whose means of computation were meagre, unless the unit of length chanced to be (as above) related in some extremely obvious way to the unit of area, as, for instance, being the side of the square which is the unit of area.] This parenthesis serves to illustrate the significance of the part