Of particles in a straight line.

Cor.—If the particles all lie in the same line, take this for Ox. Then, every y being zero, $\bar{y}$ is so also, and the centre of gravity is in Ox, its distance from O being given by

$$\bar{x} = \frac{\Sigma (w\, x)}{W}$$

The following independent proof of this may be noted.

53. Let Ox be the line in which the particles lie, O being any point from which the distances of the particles are known, and let this line be placed horizontal. Let x be the distance from O of the particle whose weight is w.

Let W be the whole weight, and $\bar{x}$ the distance of the centre of gravity from O.

Draw another horizontal line from O perpendicular to Ox. This line will then be perpendicular to the direction of the weights, and the moment about it of the whole weight collected at the centre of gravity will be equal to the algebraic sum of the moments of the several weights. Hence we have

$$W.\bar{x} = \Sigma\,(w.x)$$

or

$$\bar{x} = \frac{\Sigma\,(w.x)}{W}$$

where Σ denotes the algebraic sum of all the products corresponding to that within the bracket. Also, if a moment be reckoned positive when the particle is on one side of O, that of a particle on the other side of O will be negative, and the difference in algebraic sign of the moments will therefore at once be indicated by considering the x's of the particles to be positive or negative according as they lie on one or the other side of O.

Heavy body suspended freely, its centre of gravity is vertically above or below the point of suspension.

54. *When a rigid body rests suspended from or supported by a fixed point, and acted on only by its weight, the vertical line drawn through the centre of gravity will pass through the point of suspension or support ; and, conversely.*

For the weight of the body may be supposed collected at its centre of gravity, and there to act vertically downwards; and the necessary and sufficient condition of equilibrium is that its moment about the fixed point must vanish, which requires that its direction shall pass through this point.