

The commensurable values of y and t which satisfy (11) and (16) are,

$$y = 2000, \quad t = 50.$$

Also, by (14),

$$A = 36000.$$

Therefore, from the second of equations (9) and from (20),

$$B = -1700000, \quad B'\sqrt{z} = -400000\sqrt{5}.$$

And $u_1u_4 = -20\sqrt{5}$. Therefore

$$\begin{aligned} x &= u_1 + u_4 + u_2 + u_3 \\ &= [-100000(17 + 4\sqrt{5}) + \sqrt{\{100000^2(17 + 4\sqrt{5})^2 + (20\sqrt{5})^2\}}]^{1/2} \\ &\quad + [-100000(17 + 4\sqrt{5}) - \sqrt{\{100000^2(17 + 4\sqrt{5})^2 + (20\sqrt{5})^2\}}]^{1/2} \\ &\quad + [-100000(17 - 4\sqrt{5}) + \sqrt{\{100000^2(17 - 4\sqrt{5})^2 - (20\sqrt{5})^2\}}]^{1/2} \\ &\quad + [-100000(17 - 4\sqrt{5}) - \sqrt{\{100000^2(17 - 4\sqrt{5})^2 - (20\sqrt{5})^2\}}]^{1/2}. \end{aligned}$$

§34. *Eighth Example.*—Let

$$x^5 - 20x^3 + 250x - 400 = 0.$$

Here $g = 2$, $k = 0$. Because g is distinct from zero, we use not the formulæ (11) and (16) as in the two preceding examples, but (37) and (38). The commensurable values of y and t which satisfy (37) and (38) are

$$y = 2, \quad t = -2.$$

Therefore, from the second of equations (6), $B = 60$. The value of azA' is obtained from (35). Thus, since $(A'\sqrt{z})(a\sqrt{z}) = azA'$, and since $a\sqrt{z} = \sqrt{y} = \sqrt{2}$, the value of $A'\sqrt{z}$ is known. Then, by the second of equations (34), $B'\sqrt{z} = 44\sqrt{2}$.

Therefore

$$B + B'\sqrt{z} = 4(15 + 11\sqrt{2}).$$

And $u_1u_4 = g + a\sqrt{z} = 2 + \sqrt{2}$. Therefore

$$\begin{aligned} x &= u_1 + u_4 + u_2 + u_3 \\ &= [4(15 + 11\sqrt{2}) + \sqrt{\{16(15 + 11\sqrt{2})^2 - (2 + \sqrt{2})^2\}}]^{1/2} \\ &\quad + [4(15 + 11\sqrt{2}) - \sqrt{\{16(15 + 11\sqrt{2})^2 - (2 + \sqrt{2})^2\}}]^{1/2} \\ &\quad + [4(15 - 11\sqrt{2}) + \sqrt{\{16(15 - 11\sqrt{2})^2 - (2 - \sqrt{2})^2\}}]^{1/2} \\ &\quad + [4(15 - 11\sqrt{2}) - \sqrt{\{16(15 - 11\sqrt{2})^2 - (2 - \sqrt{2})^2\}}]^{1/2}. \end{aligned}$$

§35. *Ninth Example.*—Let

$$x^5 - 5x^3 + \frac{85}{8}x - \frac{13}{2} = 0.$$

Here $g = \frac{1}{2}$, $k = 0$. Because g is distinct from zero, we use the formulæ (37)