

by No. 3, keeping each side of equation by itself, etc. *Ans.*  $x = \frac{1}{2}$ .

(b) Either add or subtract the equations,  
etc. *Ans.*  $x = \frac{cp+aq}{ac-bd}$ ;  $y = \frac{aq+bp}{ac-bd}$ .

3. Resolve into factors:

(a)  $a^2 + b^2 - c^2 - d^2 - 2ab + 2cd$ .

(b)  $4x^4 + 6x^3 + 72x - 576$ .

(c)  $x^3 - 7x^2 - 144$ .

3. (a) *Ans.*  $= (a-b-c+d)(a-b+c-d)$ .

(b) Write  $4(x^4 - 144) + 6x(x^2 + 12)$ , etc.  
*Ans.*  $= 2(x^2 + 12)(2x^2 + 3x - 24)$ .

(c) *Ans.*  $(x-2)(x+2)(x^2+4)(x^2+9)$ .

4. Solve the equations:

(a)  $x + \frac{5}{2x} = 3\frac{1}{2}$ .

(b)  $3x^2 - 2x + \sqrt{3x^2 - 4x - 6} = 18 + 2x$ .

4. (a) *Ans.* 2 or  $\frac{1}{2}$ .

(b) Transpose  $2x$  from right to left, then diminish each side by 6; thus  $3x^2 - 4x - 6 + \sqrt{3x^2 - 4x - 6} = 12$  . . . . (2) place  $\sqrt{3x^2 - 4x - 6} = a$ , then substitute thus,  $a^2 + a = 12$ , . . . . (3) Solve this quadratic, and substitute value of  $a$  in (2). *Ans.*  $x = 3$  or  $\frac{1}{3}$ .

5. Shew that

(a)  $\frac{b-c}{1+bc} + \frac{d-a}{1+ad} = 0$ , if  $\frac{a-b}{1+ab} + \frac{c-d}{1+cd} = 0$ . (b) If  $x = \frac{b-c}{a}$ ,  $y = \frac{c-a}{b}$ ,  $z = \frac{a-b}{c}$ , then  $xyz + x + y + z = 0$ .

5. (a) Suppose  $\frac{b-c}{1+bc} + \frac{d-a}{1+ad} = 0$ , then  $b+abd - c - ucd + d + bcd - a - abc = 0$ . If  $\frac{a-b}{1+ab} + \frac{c-d}{1+cd} = 0$ , then  $a+acd - b - bcd + c + abc - d - abd = 0$ . Since this latter is zero we may subtract it from the former without changing the value, and upon doing so we find the former is zero.

(b) In  $xyz + x + y + z = 0$ , substitute the values given for  $x, y, z$ , etc.

6. (a) Solve the equation,  $ax^2 + bx + c = 0$ , and thence find the conditions for equal roots.

(b) If the roots of  $x^2 - px + q = 0$  are  $a$  and  $\beta$ , and if  $a^2 = q\beta$ , find the value of  $a$  in terms of  $\beta$ .

6. (a) Book-work.

(b) Since  $a$  and  $B$  are the roots  $\therefore (x-a)(x-B) = 0$ , or  $x^2 - x(a+B) + aB = 0$ .  $\therefore$  In the original equation  $(a+B) = p$ ,  $aB = q$ , and  $a^2 = qB$  (hypo).  $\therefore a = p-B$ ,  $a = \frac{q}{B}$ .  $\therefore \frac{q}{B} = p-B$ .  $\therefore q = pB - B^2$ ,

But  $q = \frac{a^2}{B}$ .  $\therefore a^2 = pB^2 - B^3$ .  $\therefore a = B\sqrt{p-B}$ .  $\therefore a = B\sqrt{a+B-B} = B\sqrt{a}$ .  $\therefore a = B^2$  *Ans.*

7. Solve the following equations:

(a)  $x+y = 5$ ,  $x^2+y^2 = 65$ .

(b)  $2x^2 - xy = 6$ ,  $2y^2 + 3xy = 8$ .

7. (a) Cube (1) Find difference between this result and (2) Substitute, etc.  $x=4$  or 1.

(b) Multiply (1) by 4, and (2) by 3, subtract results, and factor, thus  $(8x+3y)(x-2y) = 0$ , etc.  $y = \pm 1$ , etc.

8. (a) If  $3x = 2b+2c-a$ ,  $3y = 2a-b+2c$  and  $3z = 2a+2b-c$ , shew that  $x^2+y^2+z^2 = a^2+b^2+c^2$  and  $xy+xz+yz = ab+ac+bc$ .

(b) Prove that  $x^4 - px^3 + qx^2 - rx + s$  divided by  $x-a$  gives  $a^4 - pa^3 + qa^2 - ra + s$  as remainder.

8. (a) Add three quantities, square result, thus we find  $x^2+y^2+z^2+2xy+2xz+2yz = a^2+b^2+c^2+2ab+2ac+2bc$ . Then multiply first and second, second and third, first and third, collect, etc.

(b) Put  $x = a$ , substitute, etc.

9. If a ship requires 40 hands, a schooner 15, and a steamer 10; if on a given day 36 vessels arrive in port manned in all by 750 men; and if the hands on the ships were sufficient to supply all the schooners and twice the number of steamers, how many vessels of each kind arrived that day?

9. Let  $x$  = number of ships,  $y$  = number of steamers,  $36-x-y$  = number of schooners, etc. *Ans.* 11, 13, 12.

10. The difference of two numbers is to the less as 4 is to 3, and their product multiplied by the less is 504; find the numbers.

10. Let  $x$  = greater.  $y$  = lesser, etc. *Ans.* 14.6.