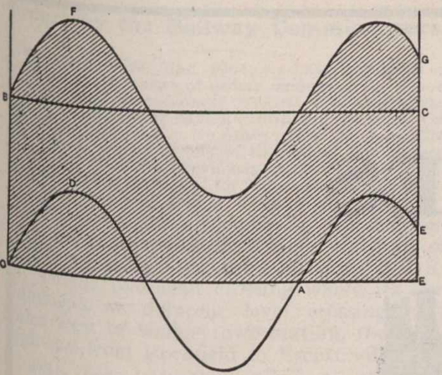

FIGURE 1

at — 37°, gradually increasing to 0.0465 ft. or about .55 inch at 75°, at which point the upward velocity is zero and the upward movement has attained its maximum. The wheel then begins to return to the rail, its movement becoming zero again, or in other words, striking the rail at 115°, and by referring to the velocity curve, it will be seen that at the point the downward velocity is 4.7 ft. per second. This velocity corresponds to that gained in dropping freely through a height of 0.36 ft., or about 4½ ins., and as the weight is 3,200 lbs., there is an actual, but not severe, blow.

It is interesting to note that this diagram explains completely the results obtained on testing plants, and with wire run under the drivers, in which W. F. M. Goss has noted that the wheel appeared to drop more quickly than it went up, and at a considerably greater distance from the centre, and it is evident that this should be so. The movement of the wheel does not coincide with the variation in the force. As long as the force is upward the wheel is acquiring an upward velocity, and this velocity does not become zero until the downward forces have acted on the wheel for a sufficient time to destroy it. In the same way, the upward movement goes on increasing, not only until the upward velocity decreases, but until it is destroyed and the wheel does not return to the track until the downward velocity has attained a very considerable amount.

The diagram in fig. 4 shows similar curves plotted at 320 revolutions for the engine mentioned as having caused the damage to the rail on the C.P.R. This is, of course, an exceptionally bad case, but it will be seen that the wheel did not return to the track for 177°, when it had a downward velocity of 17½ ft. per second, corresponding to a free fall of 4¾ ft., from which height a weight of 3,200 lbs. would certainly deliver a blow of sufficient energy to account for the effects observed.

In an extreme case of this nature, however, the method of analysis employed gives results that are greater than would actually occur, since the force acting down on the wheel is not constant, but would increase as the wheel moved upward and deflected the spring. For instance, if the latter had a deflection of 0.2 ft. under the working load of 18,800 lbs., the downward force with any upward movements of the wheel would equal

$$3200 + 18800\left(\frac{s+0.2}{0.2}\right)$$

in place of a constant amount of 22,000 lbs., and the acceleration equation would then become

$$\frac{d^2s}{dt^2} = C\cos kt - \left[3200 + 18800\left(\frac{s+0.2}{0.2}\right)\right]$$

This expression involves $s$ and becomes exceedingly complicated to integrate, but the effect of including it would be to diminish the upward movement and slightly reduce the striking velocity. In the first case its influence is inappreciable as the upward movement is small, but in the second it would certainly reduce this, and account for the box not striking the frame. An exact solution would in addition allow for the elasticity of the track, and this in its turn would apparently increase the velocity of the blow, although an equation involving it would probably be too complicated to treat mathematically except by an expert.

While, however, the solution here given in fig. 4 may not be exactly correct, the actual striking velocity being lower than that calculated, there is no doubt that it is of considerable magnitude, and probably from 12 to 15 ft. per second, and an absolute hammer blow is therefore accounted for which is of sufficient intensity to explain the damage that has occurred.

It is interesting to note that in extreme cases the wheel does not return to the track or the blow occur until the wheel has moved to a position where the counterbalance is within 20° or 30° of being vertically downward, and the popular connection of this blow with the downward movement of the counterbalance is thus explained.

The result of these calculations would emphasize the danger of an unbalanced force which could equal the weight on the wheel. On the usual assumption that the maximum speed in miles per hour equals the diameter of drivers in inches, this would restrict the overbalance in any wheel to 2½ per cent. of the weight on the wheel, and to be entirely safe the practice on the C.P.R. is now to limit it to 1¼%, and to make it 1% if possible.

MATHEMATICAL ANALYSIS.

When counterbalance is vertically upward $t = o$

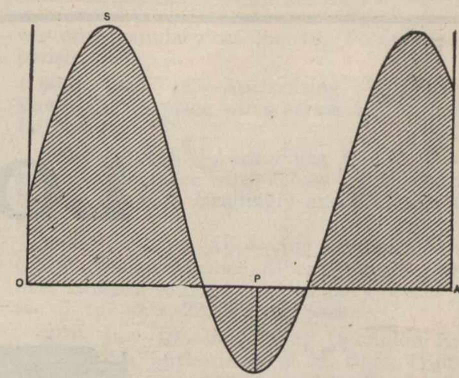

FIGURE 2

Let $w$ = acceleration due to downward force of spring and weight of wheel, acting on mass of wheel

$c$ = acceleration due to maximum value of force caused by overbalance, acting on mass of wheel

$s$ = vertical movement of wheel, from rail, feet

$t$ = time, seconds

$kt$ = angular movement of wheel, radians

Then, $\frac{d^2s}{dt^2} = c\cos kt - w$

$$\frac{ds}{dt} = \frac{c}{k}\sin kt - wt + C$$

$= o$ when $t = -t_1$, when $\cos kt_1 = \frac{c}{w}$

Then $C = \frac{c}{k}\sin kt_1 - wt_1$

And $\frac{ds}{dt} = \frac{c}{k}\sin kt + \frac{c}{k}\sin kt_1 - wt - wt_1$,

from which the velocity curves are plotted

$$s = -\frac{c}{k^2}\cos kt + \frac{ct}{k}\sin kt_1 - \frac{wt^2}{2} - wtt_1 + C$$

$= o$ when $t = -t_1$

Then $C = \frac{ct_1}{k}\sin kt_1 + \frac{c}{k^2}\cos kt_1 - \frac{wt_1^2}{2}$

$$\text{and } s = \frac{c}{k^2}(kt_1\sin kt_1 + \cos kt_1) - \frac{wt_1^2}{2} - \frac{wt^2}{2} + t\left(\frac{c}{k}\sin kt_1 - wt_1\right) - \frac{c}{k^2}\cos kt_1$$

from which the space curves are plotted.—American Engineer and Railroad Journal.

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The Life Underwriters' Association of Canada recently applied for the same rates of fares on railways in Eastern Canada as are given to commercial travellers. The application was refused in view of the different conditions of the two organizations.

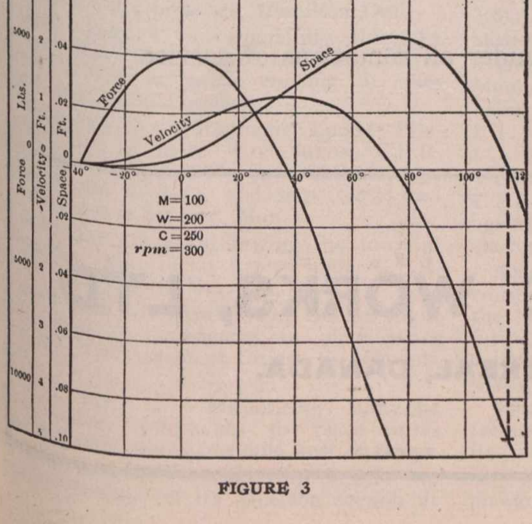


FIGURE 3

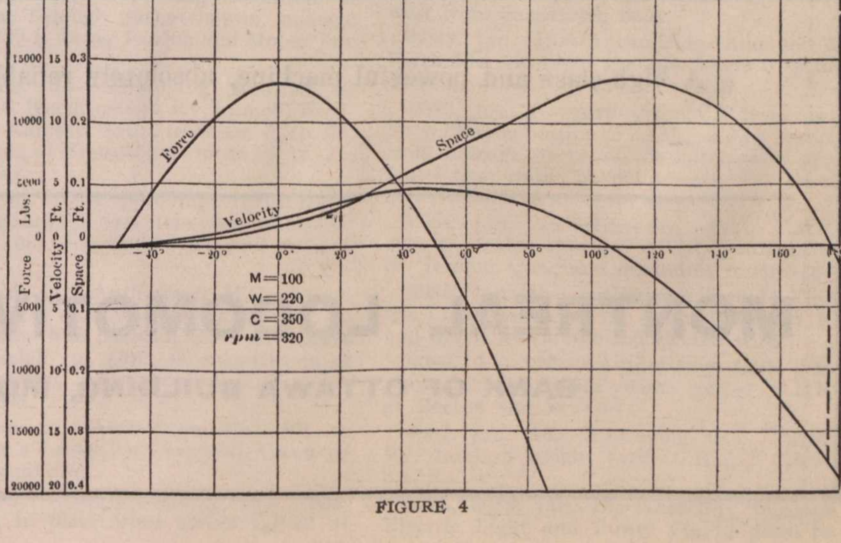


FIGURE 4