By giving to r the values 1, 2, ... n, we obtain the coefficients of all the terms after the first. Hence we have

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{1.2}a^{n-2}x^2 + \dots\dots$$

$$+ \frac{n(n-1)\dots\dots(n-r+1)}{1.2\dots\dots r}a^{n-r}x^r + \dots\dots + x^n,$$

and the required rule for the expansion has been found.

Cor. 1. The sum of the coefficients in the expansion of $(a+x)^n$ *is* 2^n. (Found by putting $a=1$, $x=1$.)

Cor. 2. The sum of the odd coefficients is equal to the sum of the even coefficients. (Found by putting $a=1$, $x=-1$.)

Cor. 3. The coefficients of terms equidistant from the beginning and the end are the same.

NOTE :—

(1) The number of terms is $n+1$, so that if n is even there is a middle term, and if n is odd there are two middle terms.

(2) For convenience the expansion is frequently written thus :

$$(a+n)^n = a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \dots\dots + \binom{n}{r}a^{n-r}x^r + \dots\dots + x^n,$$

where $\binom{n}{r}$ denotes $\frac{n(n-1)\dots\dots(n-r+1)}{1.2\dots\dots r}$. We may agree to denote the first and the last coefficient, namely 1, by $\binom{n}{0}$.

(3) $(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \dots\dots + \binom{n}{r}x^r + \dots\dots + x^n$.

(4) $(a-x)^n = a^n - \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 - \dots\dots + (-1)^r\binom{n}{r}a^{n-r}x^r + \dots\dots + (-1)^n x^n$.

Ex. 1. Find the middle term of $(2x-3y)^{10}$.

There are in all 11 terms so that the middle term is the 6th, *i.e.*, the term involving y^5. It is therefore

$$\frac{10.9.8.7.6}{1.2.3.4.5}(2x)^5(-3y)^5, \text{ or } -252.32.243x^5y^5.$$