by what is *convenient* than by what is true, and that perhaps some other more convenient explanation might some day replace the one now adopted. But in the case of the negative index such a mode of expression is still less admissible, because the steps by which the meaning is established are so easy and straightforward.

If any operation performed on a quantity x be denoted by $f^1(x)$, we should denote the same operation performed upon $f^1(x)$ by $f^1(f^1(x))$ or conveniently by $f^2(x)$. $f^2(x)$, therefore, denotes the operation f^1 performed *once* upon $f^1(x)$, or *twice* successively on x. Similarly $f^3(x)$ may be used to denote the function f^1 performed *once* on $f^2(x)$, *twice* successively on $f^1(x)$, or *three times* successively on x, and so on. Adopting this notation we shall have $f^m(x)$ to represent the operation f^1 performed m times on x successively, and $f^{m+n}(x)$ or $f^{n+m}(x)$ to represent either the performance of the operation f^1 m times on $f^n(x)$, *i.e.*, $=f^m(f^n(x))$ or n times on $f^m(x)=f^n(f^m(x))$ or $m+n$ times on x, the result being in each case the same, *i.e.*,

$$f^{m+n}(x)=f^n(f^m(x)) \qquad (\alpha)$$
$$=f^m(f^n(x)) \qquad (\beta)$$

Hence $f^m(x)$ is derivable from $f^{n+m}(x)$ by *undoing* the n operations denoted by f^n in (α) and $f^m(x)=f^{\overline{m+n}-n}(x)$.

Hence $-n$ in the index must be regarded as undoing the operation f^1 n *times* supposing it had been performed *more than n times* on x.

But what does $f^0(x)$ or $f^{-n}(x)$ represent of itself, when there is no operation to undo?

Now we observe that f^1 denotes an operation performed *once*, f^2 *twice*; f^m *m times*.

$\therefore f^0$ represents the operation performed *no* times, that is, *not performed at all*, or $f^0(x)$ is the same as x, for just as truly as f^m represents m operations, so truly does f^0 represent no operations: