152. Solve $2x^3 - x^2 = 1$.

Transpose x^2, add -2 and factor

$$x = 1 \text{ or } \tfrac{1}{4}(-1 \pm \sqrt{-7}).$$

153. Solve

$$27x^2 - \frac{841}{3x^2} + \frac{17}{3} = \frac{232}{3x} - \frac{1}{3x^2} + 5.$$

Multiply both sides by 3, transpose $\frac{841}{x^2}$ and $\frac{1}{x^2}$, add 1 to each side to complete the square

$$x = 2, \ -\frac{14}{9}, \text{ or } \frac{1}{9}(-2 \pm \sqrt{-266}).$$

154. Two inclined planes are placed so as to have a common vertex. Two weights, one on each plane, are in equilibrium when connected by a cord that passes over this common vertex; shew that the weights are to one another as the lengths of the planes on which they rest.

Let W and W_1 represent the weights and θ and θ_1 the inclination of the plane, also t be the tension of the cord, constant throughout

$$\left.\begin{aligned} t &= W \sin\theta \\ t &= W_1 \sin\theta_1 \end{aligned}\right\} \therefore W \sin\theta = W_1 \sin\theta_1,$$

the sides are as the sines of the angles opposite to them; therefore, &c.

155. Two right cones have the same base and the vertices in the same direction, but they are of different altitudes; find the distance of the centre of gravity of the solid, contained between their two surfaces, from the common base.

Let b = the base, h and h' heights of the greater and less cone respectively, v and v' their volumes.

$v = \frac{1}{3}bh$ and $v' = \frac{1}{3}bh'$, space enclosed $= v - v'$ $= \frac{1}{3}bh - \frac{1}{3}bh'$.

Let x be the distance of the centre of gravity of this enclosed space from base. Centre of gravity of a right cone is $\frac{1}{3}$ of distance along altitude from base;

$\therefore\ v \times \frac{1}{3}h = v' \times \frac{1}{3}h' + (v - v')x.$ Substitute

$$x = \frac{v\,\frac{1}{3}h - v'\,\frac{1}{3}h'}{v - v'} = \frac{\frac{1}{3}bh \times \frac{1}{3}h - \frac{1}{3}bh' \times \frac{1}{3}h'}{\frac{1}{3}bh - \frac{1}{3}bh'}$$

$$= \frac{\frac{1}{9}b(h^2 - h'^2)}{\frac{1}{3}b(h - h')} = \tfrac{1}{3}(h + h'), \text{ i.e., } \tfrac{1}{3} \text{ sum of}$$

altitudes.

Solutions by the proposer, Prof. EDGAR FRISBY, M.A., Washington.

156. Prove that $\sin 54^\circ - \sin 18^\circ = \frac{1}{2}$.

This can be done by finding the value of each and subtracting $\frac{1}{4}(\sqrt{5} + 1)$ and $\frac{1}{4}(5 - 1)$, but I prefer this:

$$\sin 54^\circ - \sin 18^\circ = 2 \sin 18^\circ \cos 36^\circ$$

$$= \frac{\sin 36^\circ}{2 \cos 18^\circ} \frac{\sin 72^\circ}{\sin 36^\circ} = \frac{1}{2}.$$

157. Prove that $x^7 - x$ is always divisible by 42.

$$\begin{aligned} x^7 - x &= x(x^2 - 1)(x^4 + x^2 + 1) \\ &= x(x^2 - 1)(x^4 - 13x^2 + 36 + 14x^2 - 35) \\ &= x(x^2 - 1)(x^2 - 4)(x^2 - 9) + 7x(x^2 - 1)(2x^2 - 5) \\ &= (x - 3)(x - 2)(x - 1)(x)(x + 1)(x + 2)(x + 3) \\ &\quad + 7(x - 1)(x)(x + 1)(2x^2 - 5). \end{aligned}$$

This is divisible by $\lfloor 7$ and the other part by $7 \lfloor 3 = 42$.

158. If α, β, γ are the distances from the centre of the inscribed circle of a triangle from its angular points, prove that $a\alpha^2 + b\beta^2 + c\gamma^2 = abc$.

$$\alpha = r \operatorname{cosec} \frac{A}{2},\ \beta = r \operatorname{cosec} \frac{B}{2},\ \gamma = r \operatorname{cosec} \frac{C}{2}$$

$$\alpha = \sqrt{\frac{(s - a)(s - b)(s - c)}{s} \frac{bc}{(s - b)(s - c)}}$$

$$= \sqrt{\frac{bc(s - a)}{s}}$$

$$a\alpha^2 = abc\left(\frac{s - a}{s}\right), \quad b\beta^2 = abc\left(\frac{s - b}{s}\right),$$

$$c\gamma^2 = abc\left(\frac{s - c}{s}\right),$$

$$\therefore\ a\alpha^2 + b\beta^2 + c\gamma^2 = abc\left(\frac{3s - 2s}{s}\right) = abc.$$

159. Having given the radii of the inscribed and circumscribing circles of a triangle and the sum of the sides, find the sides.

$$r = \frac{\Delta}{s}, \quad R = \frac{abc}{4\Delta}, \quad \therefore\ 4rRs = abc,$$

$$\begin{aligned} sr^2 &= (s - a)(s - b)(s - c) \\ &= s^3 - (a + b + c)s^2 + (ab + ac + bc)s - abc \end{aligned}$$