But though experience may be the *occasion* of our knowledge, as furnishing the materials of thought, and calling into activity the latent energies of the mind, does it follow that all our knowledge is derived from experience, uninfluenced by the creative laws of the "INTELLECTUS IPSE?"

Nay, the possibility of experience itself depends on the validity of certain cognitions which must be the property of the mind alone. For from what source, independent of the universal laws of the intellect, can these principles spring, which recognize the validity of experience? They cannot be derived from the fortuitous teaching of that experience whose trustworthiness they affirm, and they must consequently be the genuine offspring of the understanding. The principles of pure Mathematics are characterized by *necessity* and *universality*—properties which cannot possibly be given in the limited and fortuitous teachings of the sensible world. These can indeed inform us that a thing *is*, but not that it *has been*, *is*, and *will be*, in consequence of the universal necessity that it MUST be, which are the clearly conceived characteristics of Mathematical truths. The laws of the sensible world may change, and phenomena occur in violation of all past experience; there can be imagined a time when the sun shall no longer rise in the eastern and set in the western heavens; but there can never come a time when two parallel lines can meet, or two straight lines enclose a space. Pure Mathematics, therefore, furnish striking instances of intuitive truths as eternal as the intellectual principle itself, and present a brilliant example of the almost limitless extent to which the mind can develop clear and abiding knowledge, independent of the laws of the external world.

2. The principles of *Applied Mathematics* are generalizations from experience, which, by the application of pure Mathematics, are elevated to the rank of universal truths that constitute the basis of sciences commonly classified with those of pure reason. The principles of pure Mathematics are *pure, a priori* truths; those of the physical sciences founded on Mathematics, contain elements of experience. The truths of pure Mathematics are absolutely universal and necessary; while the fundamental principles of Applied Mathematics are universal only on the assumption of permanence in the operation of nature's laws. Of the former we are assured that they must be, of the latter we know only that they MUST BE *under the present condition of things*. To this extent these principles are universal and necessary, and by the aid of those of pure Mathematics, have become the elements of sciences which, as I have already said, are classed as rational. Now, *Applied Mathematics* affords a striking illustration of the application of pure mathematics to the varied phenomena of the Universe, and exhibit in a remarkable degree the harmony of deduction and induction as methods of investigation, as well as the power of the former to aid the latter in bringing the contingent and variable within the compass of universal laws. These laws are sometimes *clearly* indicated by a few well observed facts and do not seem to require the aid of pure science to demonstrate their universality. But even in these instances, Mathematical investigations frequently indicate the existence of phenomena overlooked by observation, and show the assumed law to be more general than mere experience had indicated, by demonstrating that certain facts considered as consequences of other and unknown agencies, are the legitimate results of the already discovered law. But in many instances the law is only faintly pointed out by the observed facts, or suggested by a fortunate anticipation of genius, and Mathematics are needed to verify its claims to be ranked as a GENERAL LAW, by submitting the deductions logically derived from it, to the test of coincidence with observed phenomena. Thus the logic of induction as applied to external nature, is eminently aided by the logic of pure Mathematics; the former often indicating laws beyond the scope of the latter, while, on the other hand, the latter gives them their character as general laws, and guides experience to results it would otherwise never reach. Hence, while pure Mathematics constitute a world of ideas independent of material forms, applied Mathematics grasp the phenomena of the material world as the tangible forms of the pure ideal, and show the harmony between the world without, and the inner world of thought. If the Mathematics of the world of mind, exhibit the "harmony of thought with thought," their application to external phenomena demonstrates the harmony of thought with existence, and secures to intelligence its conquest over matter. We can thus understand the importance of the knowledge given to the mind by the application of Mathematics to the investigation of external phenomena—a knowledge which alone can dispel the mists that hide the treasures of nature from our intellectual vision—which wrests the universe from the grasp of an inexorable fate, and with reverence places it before the THRONE OF GOD.

3. *Mathematical Principles and Propositions are Synthetic—leading to New Truths.* It has sometimes been said that Mathematical propositions are merely analytic—that they only resolve conceptions into their contained elements, and consequently furnish the mind with no new truths. It may be admitted that there are certain axioms and definitions in Mathematics which are identical propositions: but most of these are only minute links in the chain of method, and are not absolutely necessary to the development of the Science. Every principle really necessary in Mathematical investigation is a synthetic proposition: the predicate is not affirmed of the subject through the principle of identity, as something necessarily given in our conception of it; but on the contrary, the predicate is added as a new attribute to the contents already cogitated in our conception of the subject. For it appears evident that Mathematics could not possibly have been developed, as they undoubtedly have been, into the most perfect of the rational sciences, by combining merely identical propositions. Even the fundamental propositions of Arithmetic are not identical propositions. For, to use an illustration from Kant, in the addition of the simplest numbers, does the simple cogitation of their *union*, evolve the conception of their *sum*? The conception of such a union can never give the required predicate, and the synthesis must be effected by intuitions supplied by external objects. It is the same with the primary truths of pure Geometry. The proposition, three straight lines may enclose a space, is not an assertion of identity. For by no *analysis* of the subject—one conception of which (straight) is merely *qualitative*, the other (line) a quantity of but *one* dimension—is it possible to evolve the conception of a quantity of *two* dimensions, and the proposition is therefore synthetic. And thus we might show that every proposition essential to the establishment of the pure Mathematical sciences, is a synthetical or augmentative proposition. If such, therefore, is the real nature of Mathematical propositions, the science itself must be augmentative, and hence appears the fairness of the assertion that as wholly *given* in its principles there is merely *explicative*,—the simple evolution of a potential into an actual knowledge—and that therefore it can be of but little worth as an exercise of the higher faculties of mind. The worth of this philosophy can be easily illustrated. Take the definition of a circle; this definition includes all its properties; one property is that the circumference is to the diameter in the approximate ratio of 3.1416 to 1: is this recondite property given in the definition, so that it is conceived by a mere "negation of thought?" On the contrary it was discovered only by a train of complicated reasoning, and it is a mere perversion of language to say that it formed part of the concept *circle*, before it had been discovered by mathematical investigation. Every science worthy of the name must proceed from certain principles, and may in one sense be said to be given in its principles; but are its highest developments therefore attained with a minimum of thought? The pure Mathematical sciences are indeed founded on principles which are immutable and given intuitively; but *their law is progress*. They proceed from their primary conceptions to the most comprehensive generalizations —from the simplest abstractions to the highest within the compass of the human intellect—and yet we are asked to believe that they do nothing more than evolve the truths *contained in* their principles! If the objection has any weight when urged against Mathematics, it is equally valid against all the rational sciences, and especially against Metaphysics. For the very possibility of mental science seems to depend on the existence of propositions similar to the *a priori* synthetic principles of pure Mathematics. It is the province of Metaphysics not to analyze conceptions of things, but like Mathematics, to proceed from the synthetic judgments of the reason, to enlarge their boundaries by combination with others, till organized knowledge takes the place of isolated principles. It is true that Metaphysics either from error in method, or from the tendency of reason to attempt the solution of the insoluble, have never attained that certainty which distinguishes mathematics. But surely the latter are not of less worth because instead of groping in obscurity on a sea of doubt, they are constantly making unerring progress on the great ocean of truth.

(CONTINUED PAGE 6.)