

word "house." We may equally define a house as a space surrounded by walls or as walls enclosing a space. We habitually use the word in both senses without any ambiguity or confusion. We speak of building a house when we really mean building the walls, and of living in the house when we mean living in the interior.

V. The greatest improvement in the modern over the ancient geometry is made in the extension of the idea of angular magnitude. In Euclid only angles less than  $180^\circ$  are considered as having any actual existence. Angular measures equal to or exceeding this limit are considered merely as sums of angles to which no visible geometric meaning is attached, and which are in fact treated as purely symbolic entities, like the imaginary quantities of modern mathematics. Some moderns have followed in his footsteps so slavishly as to actually apprise the pupil that an angle of  $180^\circ$  is not an angle ! lest the pupil might be led into the mistake of considering the sum of two right angles as having some conceivable meaning !

We have already mentioned the failure of Euclid to give any definition of the sum of two angles. Without such a definition we do not know what the sum of two angles is. With such a definition the sum of two right angles becomes the angle formed by two straight lines extending from the same vertex in opposite directions.

In modern geometry angular measure is unlimited, and a given angle may have any number of measures differing from each other by any entire number of circumferences. It is not, however, advisable to burden the beginner by attempting to impress this idea upon his mind, but he should be led up to it gradually. Hence in commencing to write the present work, the author started out by confining angular measures to the limit of  $180^\circ$ . He soon found, however, that confusion would result from attempting to keep within this limit, especially in considering the relation of angles inscribed in a circle. He therefore adopted the plan of extending angular measures to one circumference, and explaining in the beginning the two measures of the angle. He finds by experience that there is no difficulty in making this double measure clear to a very young beginner.