

2. Observe the symmetry. If a is a factor the whole expression must vanish when that factor vanishes. Put $a=0$ and we get $(b+c)bc - bc(b+c)=0$. Hence a is a factor; by symmetry b and c are factors. abc is a factor.

Express $= Kabc$ where K is a numerical factor, since each term is of only three dimensions. To find K , put $a=b=c=1$, and $K=1$.

See TEACHERS' HANDBOOK p. 85.

3. For $2y$ write m and $A=x^3-mx^2+m^2x-m^3$, $B=x^3+mx^2+m^2x+m^3$.

Then $A+B=2x(x^2+m^2)$, $A-B=2m(x^2+m^2)$. x^2+m^2 , i.e. $x^2+4y^2=H.C.F.$

$$R=(a+b)(a+b-c)(a+b+c)$$

$$S=(a+b-c)(a-b+c)(a+b+c)(b+c-a)$$

$$\therefore L.C.M.=(a+b)(a+b-c)(a+b+c)(a-b+c)(b+c-a)$$

4. (1) Numr. of sum $= (x+a)(x+b)(a-b) + (x+b)(x+c)(b-c) + (x+c)(x+a)(c-a)$.

To factor this, put $(a-b)=0$ i.e. $a=b$, (as in 2) and we have $0 + (x+a)(x+c)(a-c) + (x+c)(x+a)(c-a)$ which $=0$. $a-b$ is a factor.

Expn. $= K(a-b)(b-c)(c-a)$. Put $x=0$, $a=1$, $b=2$, $c=3$ and we find $K=-1$ which is the value of the sum, since $(a-b)(b-c)(c-a)$ cancels out of Numerator and Denominator.

See TEACHERS' HANDBOOK p. 53.

(2) Put $a=0$ and we have $2s=b+c$, also

$$s(s-b)+s(s-b)(s-c)+s(s-c)(s-b)+s(s-b)(s-c)$$

$$\text{or } s^2\{s-b\} + (s-c)\{s-b\}, \text{ i.e. } s^2\{2s-(b+c)\}, \text{ which } =0.$$

Hence a is a factor and expression $= Kabc$. To find K , put $a=b=c=2$. $s=3$, and K is found to equal 1. expression $= abc$.

5. (1) Transpose thus $\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x-2}$. Add the sides sep'ly

$$\text{and } \frac{1}{x(x-1)} = \frac{-2}{x(x-2)} \therefore x=0 \text{ and } \frac{1}{x-1} = \frac{-2}{x-2} \therefore x=\frac{4}{3}.$$

(2) For $a+b$ write m and we get

$$\frac{x}{m-c} + \frac{x}{m+c} = \frac{m}{(m+c)(m-c)} \therefore 2mx = m \text{ and } x=\frac{1}{2}.$$

(3) Clear of fractions; $x(c-a+a-b+b-c)=1$

$$x(0)=1, x=\frac{1}{0}=\infty.$$

6. a shares cost $\$b$, one share cost $\frac{\$b}{a}$. Ad. of $p\% = (1 + \frac{p}{100}) \frac{b}{a}$

$\therefore c$ shares sold for $\frac{bc}{100a} (p+100)$.

Total proceeds need to be $\$b(1 + \frac{3p}{100}) = \frac{b}{100} (3p+100)$

$\therefore$ remaining $(a-c)$ shares must sell for

$$\frac{b}{100} (3p+100) - \frac{bc}{100a} (p+100)$$

$$\text{or price of one share} = \frac{b}{100(a-c)} \{3p+100 - \frac{c}{a}(p+100)\}$$

7. Taking square root of first four terms we have

$$\frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} + \sqrt{x} + \sqrt{y} \text{ the square of which agrees with the given}$$

expression except that it would require the coefficients of 6th 7th, 8th and 9th terms to be 2 instead of one. Hence there is a remainder

$$-\frac{x}{\sqrt{y}} - \frac{y}{\sqrt{x}} - \sqrt{x} - \sqrt{y}. \text{ See TEACHERS' HANDBOOK p. 11}$$

$$(1+x)^{\frac{p}{q}} = 1 + \frac{p}{q}x + \frac{p(p-q)}{1 \cdot 2 \cdot q^2}x^2 + \&c. \text{ by Binomial Theorem.}$$

$$\text{Thus } (a^3+x^3)^{\frac{1}{3}} = a(1 + \frac{x^3}{a^3})^{\frac{1}{3}} = a\{1 + \frac{1}{3}(\frac{x^3}{a^3}) + \frac{1 \cdot -2}{1 \cdot 2 \cdot 3^2}(\frac{x^3}{a^3})^2 + \&c.\}$$

$$= 1 + \frac{x^3}{3a^2} - \frac{x^6}{9a^5} + \&c.$$

8. (1) Let $a^2 + \frac{1}{a^2} = m$

$$\therefore m^2 - 2 = a^4 + \frac{1}{a^4} \text{ which } = a^4 + \frac{1}{a^4} + 2 = m + 2$$

$$\therefore m^2 - m - 4 = 0, m = \frac{1}{2}(1 \pm \sqrt{17}) = a^2 + \frac{1}{a^2}$$

Multiply 1st by a and 2nd expanded, three terms cancel and we get $y^2(c_1 - ab) + z^2(b_1^2 - ac) + yz(b_1c_1 - aa_1)2 = Ay^2 + Byz + Cz^2$

$\therefore A = c_1 - ab$, $B = b_1^2 - ac$, $C = b_1c_1 - aa_1$ if the expression is an identity.

9. (1) $2x+y=8xy=\frac{4}{3}$ from 2nd; $6x+3y=4$, $y=\frac{4-6x}{3}$

$$\therefore 6xy=8x-12x^2=1, 12x^2-8x+1=0 \text{ or } (6x-1)(2x-1)=0,$$

$$x=\frac{1}{6} \text{ or } \frac{1}{2} \&c.$$

(2) Put $y=Kx$ and divide 1st by 2nd $1+K^2=K^3+K$

$$\text{or } K^3 - K^2 + K - 1 = 0 \text{ i.e. } (K-1)(K^2+K+1)=0$$

$$\therefore K=1 \text{ or } \frac{1}{2}(-1 \pm \sqrt{-3}).$$

Substitute these values and we get three values each for x and y .

(3) Add, and $x(x+y) + (x^2+y^2) + (x+y)=0 \therefore x+y=0$ $x=-y$

Substitute this value of y in 1st and

$$-x^3 - x^2 + x = 0 \therefore x=0=y, \text{ also } -2x^2+1=0, 4x^2=2, x=\pm\frac{1}{2}\sqrt{2}$$

$$\therefore y = \mp\frac{1}{2}\sqrt{2}.$$

10. We have $x+y+z=xyz$, $2x+2z=y$, $xy-xz=$

From 3rd and 2nd $xy=x+z=\frac{1}{2}y \therefore y=0, x=\frac{1}{2}, z=-\frac{1}{2}.$

NATURAL PHILOSOPHY.

TIME—TWO HOURS AND A HALF.

1. What power (in pounds) is required to maintain a barrel weighing 150 pounds on an inclined plane. Plane inclined at an angle of 45° to the horizon.

2. A power $=6$ pounds, applied at the end of a bar of metal of uniform thickness and density, balances a weight $=64$ pounds, applied 4 inches from the fulcrum. The bar acts as a lever of the 1st class, and is 3 feet 6 inches long. Determine the pressure exerted by the system upon the fulcrum.

3. State the conditions of equilibrium of floating bodies.

4. Distinguish between:—

(i.) Mass and Weight;

(ii.) Density and Specific Gravity.

5. What pressure must be exerted upon a cylinder of fir wood, the volume of which $=94.248$ cubic inches, that it may be totally submerged in water? (Weight of cubic inch of water $=252.456$ grains.)

6. Explain the theory of the Siphon.

7. How may the centre of gravity of a body be determined experimentally?

8. A cylinder of wood 10 inches high sinks to a depth of $5\frac{1}{2}$ inches in water, to the depth of 7 inches in another liquid. What is the specific gravity of the latter liquid?

9. The diameter of the plate of a hydrostatic bellows is 16 inches, a weight of 180 pounds is placed upon it; what will be the height of the water in the pipe? Diameter of pipe one inch.

SOLUTIONS.

1 The direction of the power is omitted in the problem.

Let AB = length of plane $=L$, BC = height $=H$, AC = base $=B$.

From AB draw KM in the direction in which the power acts, take KM to represent the power, draw MN vertically downwards; and from K draw KN perpendicular to plane meeting MN in N . Then the triangle MKN represents the three forces which are in equilibrium, $\therefore P:W::R=KM:MN::NK$. But we cannot solve the triangle KMN unless the angle MKB is given, and when given we require trigonometry unless it be 30° , 45° , or 50° .

In case the power acts parallel to the plane the above proportion becomes, by similar triangles, $P:W::R=H:L::B$; if parallel to base $P:W::R=H:B::L$. And in this problem $W=150$ and $H=B \therefore L=H\sqrt{2}$, so that we have $P:150=1:\sqrt{2}$, or $P:150=1:1 \therefore P=75\sqrt{2}$ or 150.

2. The direction of the power is not given. The position of the weight is left uncertain. If it is not at the end, and the bar projects beyond the point where the weight is suspended, the problem is indeterminate. Assuming the power to act vertically and the weight to hang from the extremity to the bar, let B = weight of bar, whose centre of gravity will be 17 inches from fulcrum.

We have $4 \times 64 = 17B + 6 \times 38$, whence $B = 1\frac{1}{3}$.

Total pressure $= 64 + 1\frac{1}{3} + 6 = 71\frac{1}{3}$.

3. Book-work.

4. The word mass is used as an abbreviation for "quantity of matter."—Todhunter.

It is found that on all bodies on the earth a pressure is exerted downwards in a vertical direction, and this pressure which is called the weight of the body is invariable at the same place for the same body at all times, whatever form, size, or position the body may be made to take.—Cherriman. We estimate the mass by observing the weight, since they are proportional to each other.

Density expresses the relation between the mass and the volume.

Specific Gravity denotes the relation between the density of a given substance and the density of a standard substance.

5. S. G. of fir wood not given. It is $=.6$.

Hence submerging pressure required $=.4$, or on given cylinder $=94.248 \times 252.456 \times .4$ grains.

6. and 7. Book-work.

8. Weight of cylinder $= 5\frac{1}{2}$ inches of water $= 7$ inches of liquid

$$\therefore 1 \text{ inch of liquid} = 5\frac{1}{2} \div 7 = \frac{1}{2} \text{ inch of water. S. G.} = \frac{1}{2}.$$