RATING OF WATTMETERS FOR THREE-PHASE SYSTEMS.

By Charles Brandeis, C.E., Montreal.

The usual method of measuring the energy in a three-phase circuit is with two wattmeters connected as shown in Fig. 1, in which W_1 and W_2 represent the two wattmeters.

In a perfectly balanced three-phase system at 100 per cent. power factor these two wattmeters will read alike, i.e., each will record exactly one-half the total load. As soon, however, as the power factor drops below this value they become divergent in their readings until 50 per cent. power factor (or the current lag equals 60 degrees) is reached. At this point, W_1 will read the total energy, and $W_2 = 0$. Below

Fig. 1.

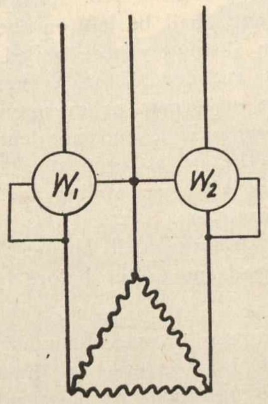


Diagram of Connections.

50 per cent. power factor the wattmeter W_2 will again indicate certain values, but they are negative, i.e., to get the total load the value of W_2 must be deducted from the value of W_1. On the other hand, above 50 per cent. power factor the sum of the two readings will give the total load. It is, of course, desirable to rate the wattmeters as low as possible for if they be too large it is not possible to read accurately when the load is small, say, at ¼ full load. Nevertheless, they must be large enough to allow for the inductive loads. To facilitate determining the correct ratings of indicating and recording wattmeters the curves devised by the writer shown in Fig. 2, will be found useful.

Let $E =$ E.M.F. of circuit,
$I =$ current.
$W_1 =$ Greater wattmeter reading,
$W_2 =$ Lesser wattmeter reading,
$W = W_1 + W_2 =$ Total watts in circuit,
$X =$ Phase angle of system.

In a balanced three-phase circuit,

$$W = W_1 + W_2 = E\ I\ \sqrt{3}\ \cos x = \text{Total power},$$
$$W_1 = E\ I\ \cos\ (x - 30^\circ)$$
$$W_2 = E\ I\ \cos\ (x + 30^\circ)$$
$$W_1 = E\ I\ \sqrt{3}\ \cos x\ \frac{1}{\sqrt{3}}\ \cos x \cos 30^\circ + \sin x \sin 30^\circ$$

$$W_1 = (E\ I\ \sqrt{3} \cos x)\ (.5 + \frac{\sqrt{3} \tan x)}{6}\ \cos x \qquad (1)$$

By a similar process:

$$W_2 = (E\ I\ \sqrt{3} \cos x)\ (.5 - \frac{\sqrt{3} \tan x)}{6} \qquad (2)$$

From this it will be observed that the wattmeter readings will be obtained by multiplying the total power in the circuit by a constant depending upon the tangent of the angle. It will also be observed that W_1 and W_2 are symetrical with reference to a horizontal axis of $+.5$. W_2 passes through zero when $(1/6\ \sqrt{4} \tan x = .5)$, or at an angle of 60°. W_1 is equal to total power expended in the circuit when $x = 60°$. The sum of the two readings represents at all times the actual power expended in the circuit. If we represent this total energy as equal to unity.

$$W_1 = .5 + \frac{\sqrt{3} \tan x}{6} \qquad (3)$$

$$W_2 = .5 - \frac{\sqrt{3} \tan x}{6} \qquad (4)$$

In practice the capacity of a machine at any given power factor is the rating of the machine multiplied by the power factor. We can derive equations representing this condition by multiplying equations (1) and (2) by cos x,

$$W_1 \cos x = (E\ I\ \sqrt{3} \cos x)\ (.5 \cos x + \frac{\sqrt{3} \sin x)}{6} \qquad (5)$$

$$W_2 \cos x = (E\ I\ \sqrt{3} \cos x)\ (.5 \cos x - \frac{\sqrt{3} \sin x)}{6} \qquad (6)$$

Assume as before, $E\ I\ \sqrt{3} \cos x = 1$

$$W_1 \cos x = .5 \cos x + \frac{\sqrt{3} \sin x}{6}$$

$$W_2 \cos x = .5 \cos x + \frac{\sqrt{3} \sin x}{6}$$

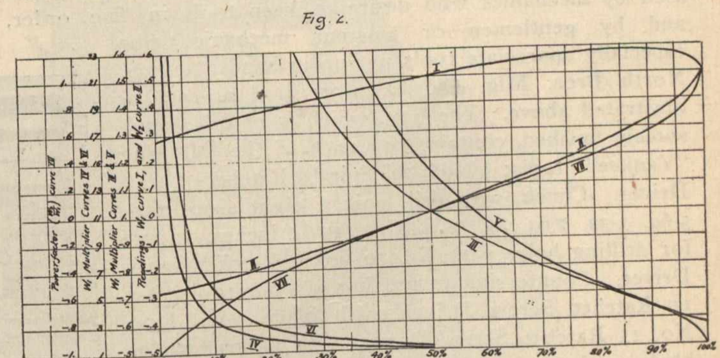
$$W_1 + W_2 = \cos x$$

If these values are plotted they give the curves I and II. These values $W_1 \cos x$ and $W_2 \cos x$ give the readings where the apparent watts in a circuit is equal to unity. In a 100-K.W. generator the volts and amperes on the line with a 100-K.W. at 100 per cent. power factor are the same as with an 80-K.W. load at 80 per cent. P.F. Consequently the values of equations (7) and (8) represent the wattmeter readings of a circuit rated non-inductively.

It is desired to find the actual wattmeter readings of a 100-K.W. machine at 80 per cent. P.F. By referring to the curves $W_1 \cos x$ (1) and $W_2 \cos x$ (2), and scale A, we find the wattmeter readings will be $.574 \times 100$ or 57.4-K.W., and $.236 \times 100$, or 23.6-K.W., a total of 80-K.W., while the apparent energy $E\ I\ \sqrt{3}$ will be 100-K.W. when the machine is loaded to its full capacity.

Multiplier curves 3 and 4, (4 continuation of 3), are derived from curve 1 by dividing the reading by corresponding power factor and enables wattmeter readings to be obtained for any given load and power factor as shown in

Fig. 2.



the following example, using scales B. and C. Actual load 50-K.W. Power factor 80 per cent. At 80 per cent. P.F. curve 3 reads .715, therefore $W_1 = .715 \times 50 = 35.75$-K.W. $W_2 = 50 - 35.75 = 14.25$-K.W.

It is now desired to find what should be the rating of the wattmeters which should be placed in three-phase circuits. This will again be determined by the apparent watts, or by the total watts which the meter will stand on a non-inductive load. It will be observed that under above conditions the volts and amperes on the generator are constant, and that the same will also be true of wattmeters placed in the circuit of the machine.

The total apparent watts on the meter in either case will be: