Let the social welfare function be:

$$W = \int_0^{\infty} B(R)\, X_0 e^{\rho t} + \int_T^{\infty} \frac{1}{2}(X_1 - X_0)\, B(R)\, e^{-\rho t} dt - sR \tag{3}$$

where

X_0 = preinvention level of output, set equal to 1,
$B(R)$ = $(C_0 - C_1)/C_0$ = unit cost saving of investment,
ρ = private and social discount rate,
T = term of the patent, and
s = cost per unit of R.

The first term of the equation (3) corresponds to the area of the rectangle C_0ABC_1 in Figure 1 above, and is the present value of the private benefit to the inventor during the patent period and the benefit to consumer after that. The second term corresponds to the triangle ABD, which is the present value of the additional gain to consumers after the patent expiry. The third term, assuming that all costs are incurred in the first period, is the present value of the cost of R&D.

Since P_0 and C_0 are preinvention price and cost, and $P_0 = C_0$; normalizing $C_0 = 1$, yields $P_0 - P_1 = C_0 - C_1 = B(R)$. From the demand equation $X_0 - X_1 = \eta(P_0 - P_1) = \eta B(R)$, where η is the slope of the demand function, representing the elasticity of demand at $P_0 = X_0 = 1$. When the expression for $X_0 - X_1$ is substituted in equation (3), the integration of equation (2) yields:

$$W = \frac{B}{\rho} + \frac{\eta}{2\rho} B^2 (1 - \psi) - sR \tag{4}$$

$$where \qquad \psi = 1 - e^{-\rho T}.$$

The inventor wants to choose the level of R&D expenditures, R, that maximizes the inventor's return or profit net of costs. The unique inventor has exclusive rights to royalties, B(R), for T periods from his process invention. The net profit maximization calculus then involves a choice of R that maximizes the present value of the royalties minus the resource cost:

$$\begin{aligned} \Pi &= \int_0^T B(R)\, e^{-\rho t} dt - sR \\ &= \frac{\psi}{\rho} B - sR. \end{aligned}$$