SOLUTIONS I.

1. Book-work.

(1.) $(a+b+c)^3 - 3(a+b+c)^2 c + 3(a+b+c)c^2 - c^3$
$= (a+b+c-c)^3$
$= (a+b)^3$

(2.) $1 - 3xy - y^3 - x^3$
$= 1 - (x^3 + x3x^2y + 3xy^2 + y^3) - 3xy + 3x^2y + 3xy^2$
$= 1 - (x+y)^3 - 3xy\{1 - (x+y)\}$
$= \{1-(x+y)\}\{1 + (x+y) + (x+y)^2 - 3xy\}$
$= (1 - x - y)(1 + x + y + x^2 - xy + y^2)$;
$\therefore$ Quotient $= 1 + x + y + x^2 - xy + y^2$.
Or the result may be obtained by $\div\, n$.

2. Book-work.

(1.) $x^3 + 6x^2 + 11x + 6$ and $x^3 + 6x^2 - 25x + 150$ have no C. M. $\therefore$ L. C. M. is their product.

(2.) $a^3 + b^3 + c^3 - 3abc$
$= a^3 + 3a^2b + 3ab^2 + b^3 + c^3 - 3a^2b - 3ab^2 - 3abc$
$= (a+b)^3 + c^3 - 3ab(a+b+c)$
$= (a+b+c)\{(a+b)^2 - c(a+b) + c^2 - 3ab\}$
$= (a+b+c)\{a^2 - ab + b^2 - ac - bc + c^2\}$
and $(a+b)^2 + 2(a+b)c + c^2 = (a+b+c)^2$;
$\therefore$ L. C. M. $= (a+b+c)^2(a^2 - ab + b^2 - ac - bc + c^2)$;
or $= (a+b+c)(a^3 + b^3 + c^3 - 3abc)$.

3. Quotient $= \dfrac{\dfrac{1+x}{1+x+x^2} + \dfrac{1-x}{1-x+x^2}}{\dfrac{1+x}{1+x+x^2} - \dfrac{1-x}{1-x+x^2}}$
$= \dfrac{(1+x^3)+(1-x^3)}{(1+x^3)-(1-x^3)} = \dfrac{1}{x^3}$

4. (a) $\dfrac{a^{3m} + a^{2m} - 2}{a^{2m} + a^m - 2} = \dfrac{a^{3m} - 1 + a^{2m} - 1}{a^{2m} - 1 + a^m - 1}$
$= \dfrac{(a^m - 1)\{(a^{2m} + a^m + 1) + (a^m + 1)\}}{(a^m - 1)(a^m + 1 + 1)}$
$= \dfrac{a^{2m} + 2a^m + 2}{a^m + 2}$

Or by finding the G. C. M. $a^m - 1$, the result is obtained by $\div\, n$

(b) $\dfrac{a(a+2b) + b(b+2c) + c(c+2a)}{a^2 - b^2 - c^2 - 2bc}$
$= \dfrac{(a+b+c)^2}{a^2 - (b^2 + 2bc + c^2)} = \dfrac{(a+b+c)^2}{(a+b+c)(a-b-c)}$
$= \dfrac{a+b+c}{a-b-c}$.

5. (1.) Let. $\dfrac{x^3 - px^2 + qx - r}{n - a} = Q + \dfrac{R}{n-a}$;

Where R does not contain x, and $\therefore$ does not change its value for a change in the value of x.
Then $x^3 - px^2 + qx - r = Q(x - a) + R$.
Put $x = a$,
and $a^3 - pa^2 + qa - r = R$;
but $a^3 - pa^2 + qa - r = 0$
$\therefore$ R $= 0$
$\therefore$ $x^3 - px^2 + qx - r$ is exactly $\div$ble by $x - a$.

(2). Put $a = 0$, and quantity becomes
$(b + c)bc - (b + c)bc$
$= 0$
$\therefore$ a is a factor,

Similarly b and c are factors, and $\therefore$ quantity is $\div$ble by abc.

To show that there is no other factor;—there can be no literal factor, for the quantity and abc are of the same dimensions. To determine the numerical factor, let $nabc$ = quantity.
Put $a = b = c = 1$;
$\therefore$ $n = 8 \times 8 - 2 \times 2 \times 2$
$= 1$;
$\therefore$ no numerical factor.

6. $\frac{1}{2}\dfrac{a^2 - b^2}{a^2 + b^2}\left(x^{\frac{1}{m}} + x^{\frac{1}{n}}\right)$
$= \frac{1}{2}\dfrac{a^2 - b^2}{a^2 + b^2}\left\{\left(\dfrac{a+b}{a-b}\right)^{\frac{2n}{n-m}} + \left(\dfrac{a+b}{a-b}\right)^{\frac{2m}{n-m}}\right\}$
$= \frac{1}{2}\dfrac{a^2 - b^2}{a^2 + b^2}\left(\dfrac{a+b}{a-b}\right)^{\frac{2m}{n-m}}\left\{\left(\dfrac{a+b}{a-b}\right)^{\frac{2n}{n-m} - \frac{2m}{n-m}} + 1\right\}$
$= \frac{1}{2}\dfrac{a^2 - b^2}{a^2 + b^2}, \left(\dfrac{a+b}{a-b}\right)^{\frac{2m}{n-m}}\left\{2. \dfrac{a^2 + b^2}{(a-b)^2}\right\}$
$= \left(\dfrac{a+b}{a-b}\right)^{1 + \frac{2m}{n-m}}$
$= \left(\dfrac{a+b}{a-b}\right)^{\frac{n+m}{n-m}}$

7. (1) $\dfrac{3 - 2x}{1 - 2x} - \dfrac{5 - 2x}{7 - 2x} = 1 - \dfrac{4x^2 - 2}{7 - 16x + 4x^2}$
$1 + \dfrac{2}{1 - 2x} - 1 + \dfrac{2}{7 - 2x} =$ etc.
$14 - 4x + 2 - 4x = 7 - 16x + 4x^2 - 4x^2 + 2$
$16 - 8x = 9 - 16x$
$7 = -8x$
$x = -\frac{7}{8}$.

(2) $3x - 2y = 1$
$\therefore$ $9x - 6y = 3$
and $5z - 6y = 1$
$\therefore$ $9x - 5z = 2$
and $7x - 4z = 1$
$\therefore$ $36x - 20z = 8$
and $35x - 20z = 5$
$\therefore$ $x = 3$;
and $y = 4$;
and $z = 5$.

(3) $\dfrac{x+3}{x+4} - \dfrac{x+1}{x+2} = \dfrac{4x+9}{2x+7} - \dfrac{12x+17}{6x+16}$;
$\therefore$ $\dfrac{(x^2 + 5x + 6) - (x^2 + 5x + 4)}{x^2 + 6x + 8}$.
$= \dfrac{(24x^2 + 118x + 144) - (24x^2 + 118x + 119)}{12x^2 + 74x + 112}$
$\therefore$ $\dfrac{2}{x^2 + 6x + 8} = \dfrac{25}{12x^2 + 74x + 112}$;
$\therefore$ $x^2 + 2x - 24 = 0$;
$\therefore$ $(x + 6)(x - 4) = 0$;
$\therefore$ $x = 4$, or -6

8. Let x = increase.
Then $\dfrac{r}{p + x} = \dfrac{r}{p} - q$
$pr = px + rx - p^2q - pqx$;
$\therefore$ $x = \dfrac{p^2q}{r - pq}$.

9. Let x = 3rd digit,
$\therefore$ $2x$ = 2nd
$9 - x$ = 1st
$\therefore$ $9 + 2x = 17$;
$\therefore$ $x = 4$;
$\therefore$ 584 = number.

10. Suppose $m > n$. Let x be the equated time. The *interest* of \$$b$ for the time $x - n$, must be $= l$ to the *discount* of \$$a$ for the time $m - x$, or

$$b(x - n)\frac{5}{100} = \frac{a(m - x)\frac{5}{100}}{1 + (m - x)\frac{5}{100}},$$

from which we obtain a quadratic $= n$ for determining x.

II.

(1) The *first* factor is at once seen to be
$8(x^2 + y^2)(y^2 + z^2)(z^2 + x^2) \div \{(x - y)(y - z)(z - x)\}^2$,
and the *second* factor is
$(x - y)(y - z)(z - x) \div (x^2 + y^2)(y^2 + z^2)(z^2 + x^2)$;
$\therefore$ result $= 8 \div (x - y)(y - z)(z - x)$.