But $\frac{\sin B}{a_3}=\frac{\sin A}{a_2}=\frac{\sin C}{a_1}$,

$\therefore p_1p_2a_3+p_1p_3a_2=p_2p_3a_1$,

or $\frac{a_1}{p_1}=\frac{a_2}{p_2}+\frac{a_3}{p_3}$.

To apply this to a case of polygon, join AC; let $AC=q$, and b_1 = perpendicular from A on AC.

Then $\frac{a_1}{p_1}=\frac{a_2}{p_2}+\frac{b_1}{q_1}$.

Similarly, $\frac{b_1}{q_1}=\frac{a_3}{p_3}+\frac{b_2}{q_2}$,

$\frac{b_2}{q_2}=\frac{a_4}{p_4}+\frac{b_3}{q_3}$,

....

$\frac{b_{n-4}}{q_{n-4}}=\frac{a_{n-2}}{p_{n-2}}+\frac{b_{n-3}}{q_{n-3}}$,

$\frac{b_{n-3}}{q_{n-3}}=\frac{a_{n-1}}{p_{n-1}}+\frac{a_n}{p_n}$;

add, and $\frac{a_1}{p_1}=\frac{a_2}{p_2}+\frac{a_3}{p_3}+\ldots\ldots+\frac{a_n}{p_n}$.

5. Prove that

$$\tan\frac{\pi}{2^{n+1}}\left\{\tan\frac{\pi}{2^{n+1}}+2\tan\frac{\pi}{2^n}+\ldots\ldots +2^{n-2}\tan\frac{\pi}{2^3}+2^{n-1}\right\}=1.$$

Since $2\cot 2A=\frac{\cot^2 A-1}{\cot A}=\cot A-\tan A$,

$\therefore \cot A=\tan A+2\cot 2A$

$=\tan A+2\tan 2A+2^2\cot 2^2 A=$etc.

$$\therefore \cot\frac{\pi}{2^{n+1}}=\tan\frac{\pi}{2^{n+1}}+2\tan\frac{\pi}{2^n}+2^2\tan\frac{\pi}{2^{n-1}}+\ldots\ldots+2^{n-2}\tan\frac{\pi}{2^3}+2^{n-1}\cot\frac{\pi}{4},$$

or $1=\tan\frac{\pi}{2^{n+1}}$

$$\left\{\tan\frac{\pi}{2^{n+1}}+2\tan\frac{\pi}{2^n}+\ldots\ldots+2^{n-1}\right\}.$$

N.B.—This problem may also be solved from the identity

$$\sin x=2^n\cos\frac{x}{2}\cos\frac{x}{2^2}\ldots\cos\frac{x}{2^n}\sin\frac{x}{2^n},$$

by taking logs of both sides, differentiating with respect to x, and putting $x=\frac{\pi}{2}$.

6. If $\frac{bx+cy+az}{cx+ay+bz}=1$, shew that

$$\frac{b-c}{cy-bz}=\text{anal.}=\text{anal.}$$

we have

$c(bx+cy+az)+abz=c(cx+ay+bz)+abz$

$\therefore abz-bcx-acz+c^2x=c^2y-acy-bcz+abz$,

i.e., $(b-c)(az-cx)=(c-a)(cy-bz)$,

$\therefore \frac{b-c}{cy-bz}=\frac{c-a}{az-cx}=$etc. in same way.

7. Solve
$x^2-yz=a$. (1)
$y^2-zx=b$. (2)
$z^2-xy=c$. (3)

$(2)\times(3)-(1)^2$ gives

$x(3xyz-x^3-y^3-z^3)=bc-a^2$, etc. $=$etc.

$\therefore \frac{x}{a^2-bc}=\frac{y}{b^2-ca}=\frac{z}{c^2-ab}=\lambda$ say.

from (1) $\lambda=(a^3+b^3+c^3-3abc)^{-\frac{1}{2}}$, whence x, y and z.

8. If n be any integer >1, shew that

$$\frac{\lfloor 2n}{\lfloor n}<\left\{8n^2(2n^2-1)\right\}^{\frac{n}{3}}.$$

We have $\frac{a_1+a_2+\ldots+a_n}{n}>(a_1a_2\ldots a_n)^{\frac{1}{n}}$,

$$\therefore \frac{n+1+n+2+\ldots+2n}{n}>\left\{\frac{\lfloor 2n}{\lfloor n}\right\}^{\frac{1}{n}}$$

or $\left\{\frac{3n+1}{2}\right\}^n>\frac{\lfloor 2n}{\lfloor n}$.

Now $\left\{\frac{3n+1}{2}\right\}^3<8n^2(2n^2-1)$, n any integer >1 for if $n=1$, the above inequality becomes $2^3=8$, but for all higher integral values of $n>1$ this inequality holds,

$\therefore$ a fortiori $\frac{\lfloor 2n}{\lfloor n}<\left\{8n^2(2n^2-1)\right\}^{\frac{3}{n}}$.

9. Examine the statement that every even number is the sum of two prime numbers, and every odd number the sum of three prime numbers.

The propositions appear to be true for all primes up to 100. We do not know of any formula expressing primes only, such as required here. The converse, however, is very obvious.