By substituting for y

$$\delta_{aa} = 16 \frac{f^2}{l^4} \int_0^l (lx - x^2)^2 \, dx + li^2$$

$$\text{or,} \quad \delta_{aa} = \frac{8}{15} f^2 l + li^2 = \frac{8}{15} f^2 l \left[1 + \frac{15}{8}\left(\frac{i}{f}\right)^2\right].$$

Introducing

$$\alpha = \frac{1}{1 + \frac{15}{8}\frac{i^2}{f^2}},$$

the equation for the line of influence of the horizontal pressure will be

$$X = \frac{5}{8} \alpha \frac{x(l-x)}{fl} \left[1 + \frac{x}{l} - \left(\frac{x}{l}\right)^2\right].$$

The last factor does not vary considerably; for $x = 0$ and $x = 1$ the value is 1 and its maximum value, reached for $x = \frac{1}{2}l$, is 1.25. No great error will therefore arise by taking this factor as a constant, which again leads to the line of influence becoming a parabola. The mean

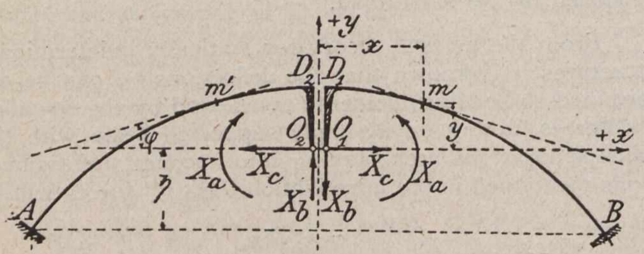

Fig. 2.

value of that factor used, is that which gives the same horizontal pressure for a uniformly distributed total load. It is calculated by either the exact equation of the approximation, whereby the equation for X attains the simple and suitable form

$$X = \frac{3}{4} \alpha \frac{x(l-x)}{lf}.$$

The line of influence of the bending moment M_m at the various points m is consequently easily derived from the general equation

$$M_m = M_{0_1 m} - M_{a_1 m} X,$$

where $M_{0_1 m}$ is the corresponding moment in the auxiliary system and $M_{a_1 m}$ is the moment produced by $X = -1$, which again is equal to the ordinate y of the point m, so that $M_m = M_{0_1 m} - yX$.

The equation for the line of influence of the normal pressure N_m in the arch is $N_m = -X$, on account of the assumed small rise of the arch.

By means of these lines of influence the moments and horizontal pressures for different points of the arch and varying loading conditions have been determined as follows:

For a uniformly distributed load g per unit length acting over the entire span the horizontal pressure will be

$$H = \frac{gl^2}{8f} \alpha;$$

and the bending moment at a point of the centre line of the arch at a distance x from one of the hinges and having the ordinate y,

$$M = \frac{gl^2}{8f} y (1 - \alpha).$$

For $\alpha = 1$, which corresponds to the case where the shortening of the arch, due to the load on it, is so small that it can be neglected, $M = 0$ and consequently the maximum and minimum moments produced by a uniformly distributed live load have the same numerical values.

For a live load p per unit length the maximum moment at the crown of the arch is

$$\text{max. } M = \frac{1}{4} pl^2 \left[\frac{1-\alpha}{2} + \alpha\left(1 - \frac{2}{3\alpha}\right)^3\right];$$

and the corresponding horizontal pressure,

$$H = \frac{pl^2}{8f} \alpha \left[1 - 2\left(1 - \frac{2}{3\alpha}\right)^2 \left(1 + \frac{4}{3\alpha}\right)\right];$$

and,

$$\text{min. } M = -\frac{1}{4} pl^2 \alpha \left(1 - \frac{2}{3\alpha}\right)^3;$$

with

$$H = \frac{pl^2}{8f} . 2 \alpha \left(1 - \frac{2}{3\alpha}\right)^2 \left(1 + \frac{4}{3\alpha}\right).$$

For $\alpha = 1$,

$$\text{max. } M = + \frac{1}{108} pl^2, \; H = \frac{13}{27} \frac{pl^2}{8f};$$

and

$$\text{min. } M = - \frac{1}{108} pl^2, \; H = \frac{14}{27} \frac{pl^2}{8f}.$$

For $\alpha = .974$, $\left(\frac{i}{f} = .12\right)$,

$$\text{max. } M = + \frac{1}{92} pl^2, \; H = .54 \frac{pl^2}{8f};$$

$$\text{min. } M = - \frac{1}{130} pl^2, \; H = .46 \frac{pl^2}{8f}.$$

The greatest possible bending moment in the arch acts at a point, the abscissa of which lies between $x = .23\,l$ and $.25\,l$, according to the value of α.

For $\alpha = 1$, the abscissa is $x = .23\,l$, and,

$$\text{max. } M = + \frac{1}{62} pl^2, \; H = .40 \frac{pl^2}{8f};$$

and,

$$\text{min. } M = - \frac{1}{62} pl^2, \; H = .60 \frac{pl^2}{8f}.$$

For $\alpha = .974$, $x = .24\,l$ and

$$\text{max. } M = + \frac{1}{58} pl^2, \; H = .43 \frac{pl^2}{8f};$$

and,

$$\text{min. } M = - \frac{1}{67} pl^2, \; H = .57 \frac{pl^2}{8f}.$$