

3. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

The sides of a triangle are cut by a circle concentric with the inscribed circle, and each vertex of the hexagon formed by the intersections is joined to the opposite vertex; prove that the triangle so formed is similar to the triangle formed by the points of contact of the inscribed circle with the sides; and if a similar construction be made with this triangle formed by the diagonals, and so on continually, the ultimate triangle is equilateral.

4. Inscribe an equilateral and equiangular quindecagon in a given circle.

Inscribe in a given regular polygon of any number of sides a regular polygon of twice the number of sides with its alternate sides coincident with the sides of the given polygon.

5. The sides about the equal angles of triangles which are equiangular to one another are proportionals; and those which are opposite to the equal angles are homologous sides, that is, are the antecedents or consequents of the ratios.

Describe a circle which shall cut three fixed right lines at given angles.

6. If a solid angle be contained by three plane angles, any two of them are together greater than the third.

A tetrahedron is cut by a plane so that the section shall be a rhombus; prove that the side of the rhombus is half the harmonic mean between a pair of opposite edges.

7. Prove that the tangents at the ends of a focal chord of a parabola intersect at right angles on the directrix.

Focal chords of a parabola at right angles to one another meet the directrix in T , t . shew that the bisectors of the angles between the tangents from either of the points T , t are parallel to the tangents from the other; and that every pair of the four tangents intersect at constant angles.

8. If from any point T in the tangent at P to a conic TM , TN be dropped perpen-

dicular to SP and the directrix, then $SM : TN :: SA : AX$.

The tangents to a conic at the ends of the latus rectum meet any ordinate PP' in Q , Q' ; shew that a circle on QQ' as diameter intercepts on the latus rectum a length equal to PP' .

9. PN is the ordinate drawn from any point P of an ellipse to the major axis AA' ; prove that

$$PN^2 : AN \cdot NA' :: BC^2 : CA^2.$$

Any point P on an ellipse is joined to A , A' ; and AF is drawn perpendicular to $A'P$. AP and AF meet the tangent at A' in K and L . Prove that $A'K$ is to $A'L$ in a constant ratio.

10. Tangents drawn from any point to an ellipse make equal angles with the focal distances.

Prove that the locus of the intersection of tangents to an ellipse which make equal angles with the major and minor axes respectively, and are not at right angles, is a rectangular hyperbola whose vertices are the foci of the ellipse.

11. Prove that if tangents at right angles can be drawn to an hyperbola they intersect on a fixed circle.

Four tangents to an hyperbola form a rectangle. If one side AB of the rectangle cut a directrix of the hyperbola in X and S be the corresponding focus, shew that the triangles XSA , XSB are similar.

12. Prove that the section of a right cone by a plane is a conic section whose foci are the points of contact with the plane of spheres inscribed in the cone to touch the plane.

If two sections of a right cone have a common directrix the latera recta of the sections are in the ratio of their eccentricities.

ARITHMETIC, ALGEBRA, AND PLANE TRIGONOMETRY.

1. A man has £1,583 17s. 11d. in 3 per cent. Stock, and £982 12s. 6d. in 3½ per cent. Stock; he transfers a certain sum from the former to the latter, when the Stocks are at 91 and 98 respectively, and thus makes