If d is the depth of the section in the plane of bending, the Inertia moment relatively to the neutral axis can be expressed in the form.

$$I = nAd^2,$$

and the section modulus in the form Sm = 2nAd. (See Appendix V. of the report).

It is desirable in pillars that there should be no tension, and generally when the vertical load is considerable there is none. Cases in which the eccentricity is so great that there is tension must be treated by the methods applicable to beams if it is made a condition that the steel carries all the tension. In the following cases it is assumed that there is no tension.

Case I.—Pillar of Circular Section, Reinforcements Symmetrical and Equi-distant from the Neutral Axis.—Let m be the modular ratio = Es/E, A the effective cross-section of the column in square inches, Av the area of vertical reinforcement in square inches, d the diameter of the pillar, dv the distance between the vertical reinforcing bars perpendicular to the neutral axis. Then the equivalent section is

$$Ae = A + (m - 1)\ Av,$$

and the section modulus is (Appendix V. of the report).

$$Sm = \tfrac{1}{8}Ad + \tfrac{1}{2}(m - 1)Av\frac{dv^2}{d}$$

The stress at the edges of the section can then be calculated by the general equation

$$f = W\left\{\frac{1}{A} \pm \frac{e}{Sm}\right\}$$

where e is the eccentricity of the load in inches and W the weight of load in lb. The greater value of stress must not exceed the safe stress stated above.

Case II.—Rectangular Section with Reinforcement Symmetrical and Equi-distant from the Neutral Axis.—Using the same notation as in the last case, d being now the depth of the section in the plane of bending, the section modulus is (Appendix V. of the report).

$$Sm + \tfrac{1}{6}Ad + \tfrac{1}{2}(m - 1)Av\frac{dv^2}{d}$$

and the stresses are given by the same equation as in the previous case.

Case III.—Column of Circular Section with Reinforcing Bars Arranged in a Circle.—Using the same notation as in Case I., ht being the diameter of the circle of reinforcing bars, the section modulus is (Appendix V.)

$$Sm = \tfrac{1}{8}Ad + \tfrac{1}{4}(m - 1)Av\frac{dv^2}{d}$$

and the stresses are given by the same equation as in Case I.

(c) Long Pillars Axially Loaded.

For pillars more than 18 diameters in length there is risk of lateral buckling of the pillar as a whole. The strength of such pillars would be best calculated by Gordon's formula, but there are no experiments on long pillars by which to test the values of the constants for a concrete or concrete and steel pillar. There does not seem, however, to be any probability of serious error if the total load is reduced in a proportion inferred from Gordon's formula to allow for the risk of buckling.

Let, as before, A = the area of the column in inches; Av = the area of vertical reinforcement. Then Ae = A + (m — 1) Av is the equivalent section. Let N be the numerical constant in the equation $I = NAd^2$ (Appendix V.), and d the least diameter of the pillar.

Then for a pillar fixed in direction at both ends Gordon's formula is

$$\frac{W}{Au} = \frac{1}{1 + \dfrac{l^2}{C_1 N d^2}} = \frac{1}{1 + C_2}$$

so that the pillar will carry less than a short column of the same dimensions in the ratio of $1 + C_2$ to 1, or, in other words, the column will be safe if calculated as a short column, not for the actual weight or pressure P, but for a weight or pressure = $(1 + C_2)$ W.

The constant C_1 has not been determined experimentally for reinforced long columns. But its probable value is

$$C_1 = \frac{4\pi^2 Ec}{u}$$

where u is the ultimate crushing stress. Putting Ec = 2,000,000 and u = 2,500, then C_1 = 32,000. Looking at the well-understood uncertainty of the rules for long columns, very exact calculation is useless. Some values of N for ordinary types of column are given in Appendix V. Taking these values, the following are the values of $1 + C_2$:

Values of $1 + C_2$.

$\frac{l}{d}$	Case I. N = 0.098	Case II. N = 0.075	Case III. N = 0.0646
20	1.13	1.17	1.19
25	1.20	1.26	1.30
30	1.29	1.38	1.44

The differences of $1 + C_2$ for considerable differences of N are not very great. In any case N can be found by the method in the Appendix with little trouble.

In the case of columns fixed at one end and rounded or unfixed at the other, $2C_2$ must be substituted for C_2. If the column is rounded at both ends, $4C_2$ must be substituted for C.

Key to the Notation.

The notation is built up on the principle of an index.

The significant words in any term are abbreviated down to their initial letter, and there are no exceptions.

Capital letters indicate moments, areas, volumes, total forces, total loads, ratios, and constants, etc.

Small letters indicate intensity of forces, intensity of loads, and intensity of stresses, lineal dimensions (lengths, distances, etc.), ratios, and constants, etc.

Dashed letters indicate ratios, such as a_2, c_2, n_1, etc., where the a, c, and n indicate the numerators in the respective ratios. The dash itself is mnemonic and is an abbreviation of that longer dash which indicates division or ratio.

Subscript letters are only used where one letter is insufficient; and the subscript letters themselves are the initial or distinctive letters of the qualifying words.

Greek letters indicate ratios and constants. They are sparingly used and are subject to the "initial letter" principle.

The symbols below are arranged in alphabetical order for facility of reference.

Standard Notation.

In Pillars.

A = the effective area of the pillar (see definition on page 326).

Ae = Area equivalent to some given area or area of an equivalent section or equivalent area.

As = cross-sectional area of a vertical or diagonal shear members, or group of shear members, in the length p, where p = pitch of stirrups.