

subject and its beneficial effect on the pupil, to become one who measures the capacity of all pupils solely by their ability to "do sums, and to work problems," and the goodness of any course of study chiefly by the work in mathematics it prescribes.

But after all has been said that can be said of the necessity of broad general culture for the teacher of mathematics, it is still emphatically true that his efficiency depends ultimately on his special training, on his resources in and power over his own subject; for it is through that subject that his special duty to the pupil is to be done. Either the pupil is to receive through him a peculiar insight into the marvellous system of the external world, some comprehension of the wonderful power and fertility of the human mind in one of its fields of activity, a quickening of his intellectual life through the knowledge and insight, the clearness and adequacy of exposition of mathematical truths characteristic of a scholarly enthusiastic, well-trained teacher; or he must forever remain indifferent to these interesting experiences, because both he and his teacher move about vaguely in a "world unrealized," where the intellectual fog never rises, and where the road traversed to-day must be re-traversed with the same dim uncertainty to-morrow.

If the teacher of mathematics has little mathematical ability, and if at the same time he has not had sufficient training, he is almost sure to carry on his work with a benumbing inadequacy of comprehension and exposition which soon becomes chronic, and through which the pupils come to look on mathematics as a highly artificial subject of little real interest or practical utility; a subject in which success does not depend on common sense and patient study, but on a certain inborn ingenuity of manipulat-

ing postulates, hypotheses, and previous propositions, and in which absurdities are as valid as realities.

A certain college student of my acquaintance must have had this kind of instruction. She said she had studied algebra before coming to college, but she had become interested in the subject only when she took up equations and proved "things." Whereupon I asked her if she had ever seen the paradox by which, through a series of equations, any number may apparently be proved equal to nothing. As she had never seen it, I showed it to her, securing her assent to the several steps as I went on. When we arrived at the conclusion,  $2 = 0$ , I handed her the paper and looked up with the half apologetic, half triumphant manner of one who expects after a very brief triumph and a rather lame defence to yield the point in question. But nothing of the sort happened. She merely said "That's all right" and handed the paper back to me. "But," I said, "how can it be? Two cannot be really equal to nothing." "O," she said, "that's algebra."

The situation is not much improved, although it is quite different, if the teacher has good ability, but insufficient training. In that case he is oppressed by the consciousness of the heavy demands made on him and of his inability for a long time to respond to them as he should. The work must be done somehow, the classes cannot wait. He is thus obliged to carry on a discouraging struggle against tremendous odds. Meanwhile, his pupils are the losers, and the reputation of the subject suffers. Similar statements could, of course, be made with respect to inadequate preparation in other subjects as well as mathematics, but the immediate consequences are not so conspicuous. The definiteness and rigor of mathematical reasoning afford a constant