

three equal groups ; what does he do ? He places *one* thing in each of *three* places, then another of the things in each of these three places, and so on till *all* the things are distributed. That is, he takes *one* of the things three *times*, then another of them three times, and so on. Finally, he counts the number of things in one of the equal groups, and his problem is solved. The example of division by an abstract divisor (*see* Nov. number CANADA EDUCATIONAL MONTHLY, p. 330) exactly represents the symbols the "taling" operations by which the child distributes his things into a number of equal groups ; and, it may be added, the examples there given are typical, both in thought-process, and symbol-process, of all ordinary operations in division. It may be added, further, that the symbols and operations in elementary mathematics should not be inconsistent with what the student is to meet with in his subsequent course. Nature makes no leaps ; neither should science. We have then $\$4 \times 3 = \12 , and therefore $\$12 \div \$4 = 3$, an easy and perfectly valid operation. But $\$12 \div 4$, the other inverse of the multiplication, is alleged to be a meaningless and impossible operation, having, of course, no relation with the former. But the student is soon to learn that $a \times b = c$, and that, therefore, $c \div a = b$; and also $c \div b = a$; both inverses intelligible, both valid, both necessary, both universal ; or all mathematical reasoning is fallacious, and mathematical science a delusion.

UNITY AND UNIT.

The fact that number as conceived by the mind is the result of the fundamental activity, analysis-synthesis, which breaks up a whole into parts relates these parts, and re-combines them into the whole, is the basis of all right method of instruction in number ; and to ignorance or neglect

of this law of mental construction, may be ascribed the numerous errors and inconsistencies in both principles and practice, to some of which reference has been made. As "a good beginning is the half of all"—especially in educational work—we may well ask, what shall be our starting point in primary number teaching ? Beginning, of course, with objects, what number of objects shall we begin with ? With *one* thing, an entity in itself, a fixed unit, as Grube and a host of followers do ? Or, with a group of things a *whole composed of parts*, as sound psychology suggests ? A simple question, yet a very important one, since upon it hangs the distinction between good method and bad, between a method which aids, and a method which thwarts the normal action of the mind. Analysis is the law under which the child's mind *must* work. What is the result of its working upon a single thing, a *one* presented as some fixed "unit" ? If there is, in this case, any analytic activity at all, it must be in the discrimination of *qualities* which make the thing, the individual, what it is. The activity cannot be in the *relating* process, which is the very essence of the conception of number. There are no objects to relate ; there is no integer or unity to be broken into parts and again reproduced from the parts. According to the normal action of the child's mind the single thing is a *unity*, but not a *unit*. That is, it has certain qualities which make it what it is ; which give it *unity* of meaning ; but it has *not* that *relation* to others of its kind which alone makes it a *unit*. The child has been making *unities*—in every act of attention—long before he begins the study of numbers ; but he has *not* been conceiving *units*. To conceive of a banana as a *unity* is simply to discriminate and unify its qualities ; to conceive it as a *unit* to think it in its relation to a number of