

$$\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$$

Given expression = $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{3}{11}$

$$= \tan^{-1} \frac{45}{11} = \frac{\pi}{4}$$

10. If

$$\frac{\tan(a+\beta-\gamma)}{\tan(a-\beta+\gamma)} = \frac{\tan \gamma}{\tan \beta},$$

prove that

$$\sin 2a + \sin 2\beta + \sin 2\gamma = 0$$

$$\frac{\tan(a+\beta-\gamma)}{\tan(a-\beta+\gamma)} = \frac{\tan \gamma}{\tan \beta}$$

∴ using componends and dividends we have

$$\frac{\tan a + \beta - \gamma + \tan a - \beta + \gamma}{\tan a + \beta - \gamma - \tan a - \beta + \gamma} = \frac{\tan \gamma + \tan \beta}{\tan \gamma - \tan \beta}$$

$$\text{or } \frac{\sin 2a}{\sin 2(\beta-\gamma)} = \frac{\sin(\beta+\gamma)}{\sin(\gamma-\beta)}$$

$$\therefore \frac{\sin 2a}{2 \sin(\beta-\gamma) \cos(\beta-\gamma)} + \frac{\sin(\beta+\gamma)}{\sin(\gamma+\beta)} = 0$$

Clear of fractions, and we have

$$\sin 2a + 2 \sin(\beta+\gamma) \cos(\beta-\gamma) = 0$$

$$\text{or } \sin 2a + \sin 2\beta + \sin 2\gamma = 0.$$

11. Eliminate θ and ϕ from the equations,

$$\tan \theta \cdot \cot \phi = \tan a \cdot \cot a_1,$$

$$\cos^2 \theta = \cos a \cdot \sec \beta$$

$$\cos^2 \phi = \cos a_1 \cdot \sec \beta$$

$$\cos^2 \theta = \cos a \cdot \sec \beta$$

$$\therefore \sec^2 \theta = \sec a \cdot \cos \beta$$

$$\text{Also } \sec^2 \phi = \sec a_1 \cdot \cos \beta$$

$$\therefore \tan^2 \theta = \sec a \cos \beta - 1$$

$$\text{and } \tan^2 \phi = \sec a_1 \cos \beta - 1$$

∴ the eliminant is

$$\frac{(\sec a \cdot \cos \beta - 1)}{\sec a_1 \cdot \cos \beta - 1} = \tan a \cdot \cot a_1$$

12. A straight line cuts three concentric circles in A, B, C, and passes at a distance p from the centre; show that the area of the triangle formed

by the tangents at A, B, C is $\frac{1}{2p} (BC \cdot CA \cdot AB)$

rect. AR.AD. = rect. CR.CD = YR.p, from which we get

$$AR = CD. \quad \text{Euc. VI. 26.}$$

$$\text{Similarly } BN = CD \text{ and } AS = BD.$$

Now area of XYZ

$$= \Delta ABX + \Delta CBZ - \Delta CAY.$$

$$= \frac{1}{2} AB \cdot XS + \frac{1}{2} CB \cdot ZN - \frac{1}{2} CA \cdot YR.$$

$$= \frac{1}{2} AB \cdot \frac{AS \cdot AD}{p} + \frac{1}{2} CB \cdot \frac{NB \cdot BD}{p} - \frac{1}{2} CA \cdot \frac{CD \cdot CR}{p}$$

$$= \frac{1}{2p} \{ AB \cdot (AC + CD) (BC + CD) + CB \cdot CD (BC + CD) - CA \cdot CD (CA + CD) \}$$

If this be expanded and coefficients of CD and CD^2 be collected, these will vanish, and there will be left

$$\frac{AB \cdot BC \cdot CA}{2p}$$

CORRESPONDENCE.

N. S., Montague Cross, P.E.I., sent eight problems in Arithmetic.

E. R., South Mountain, sent three questions in Mensuration.

M. J. S., P.E.I., sent a partition problem.

A SUBSCRIBER, Montreal, sent a problem.

M. Y. TAYLOR, Stewartville, very kindly pointed out an error in the first line of the first solution given on p. 154, Oct. 1, 1894. It reads 4886 bush. 36 lbs. wheat @ 58c, \$2,854.76 instead of \$2,834.33, as it plainly should be; and the total will then be \$2,916.46. We are much obliged for the correction, and hope that all similar mistakes will be promptly rectified by our correspondents who may detect them. This friend also sends a solution of No. 3, Ex. xxiv., p. 84, P. S. Arith., and gets the answer \$20.90 by taking in only one-half the area of the doors and windows. She wishes to know the reason of the rule calling for this deduction. "Use and wont" seem to be the only reason, for, on consulting several good text-books, we find no mention of such rule. Can any person give a better reason? Do workmen actually follow this rule in computing the price of their work? Is it a

local arbitrary regulation, or is it universally acknowledged? We can find no trace of it elsewhere.

E. J. D. sent a clock problem, which has been solved in this column before.

M. MILLER sent a discount problem, which has been solved in this column before.

THOS. L. BUCKTON, Colchester, Ont., solved Nos. 1, 3, 5, 6, 7, 8. Many thanks.

A. B. CHALMERS, Milverton, sent two solutions of No. 112, one analytic, the other deductive, both neatly set down. He also sends a problem for solution, and says some very kind things about THE JOURNAL.

F. MCE., Orchardville, solved 102, 103, 104 105, 106, 107, 108, and 109 of December issue. Very good for a girl only 14 years old.

E. J. DAVEY, Glanford, sends a solution of No. 112.

P. J. B., Montague Cross, P.E.I., sends nine problems in arithmetic.

J. S. THOMAS, Waterloo, Ont., solved 102, 103, 104, 105, 106, 107, 108, 109, and 112, and remarks that "THE JOURNAL is a grand help to teachers." Such practical help and such words of appreciation are highly prized by THE JOURNAL staff. All honor to those who are ardently desirous to help forward the teachers' cause in Canada!

W. R. B. sends an interesting letter, which shows that he is attempting to master mathematics under a load of difficulties. Courage, comrade! "Time and patience turn the mulberry leaf into satin."

J. S. MCNAMARA, Penetanguishene, sends neat concise solutions of 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, and 112. We present him sincere thanks on behalf of the thousands of readers who will be helped by the work of such correspondents as our generous friend, J. S. MCN.

A FRIEND, who gives no signature, sends solutions to 102, 103, 104, 105, 106, and 109, and they are good solutions, too.

There are a number of letters of a more private character in the Editor's pigeon-hole, and one or two more that deserve public acknowledgment and thanks.

J. E. HOLT, Newton Robinson, asks whether the rules found in the *Pub. Sch. Arith.* for the solidity of a frustrum of a cone or of a pyramid, and also for the solidity of a wedge, are absolutely correct or merely approximations. *Ans.*: Absolutely correct. He asks for answers to other questions about the same charming volume. These will receive attention at the earliest moment possible. The pressure on this column is increasing beyond the limits of our space. Will all our friends please exercise a little patience? The stack of correspondence that has accumulated would fill one whole issue of THE JOURNAL from cover to cover, and still leave enough to occupy our allotted space for another number. J. E. H. and all the other kind friends who are working to make THE JOURNAL a valuable help, and to extend its influence to every school in Canada, are working in a very effective way to raise the status of the whole profession. United solidly in confraternity we can accomplish a good deal in many ways, and "the sympathy of numbers" is a great power to encourage every true worker. Let us stand together and labor for the common good through the friendly medium of our own paper. . . . We give up this month to the Scholarship Problems. Mr. Cornwell has very courteously supplied neat solutions.

Examination Papers.

EAST SIMCOE PROMOTION EXAMINATION.

December 6th and 7th, 1894.

GRAMMAR—THIRD CLASS.

Juniors will do any five questions; seniors any six questions. Questions having equal value, 15 and 12½. Values—75 marks.

I.—Define with examples: Phrase, Simple Sentence, Predication, Preposition, Modifier.

II.—Analyze fully: (a) On the road we saw a small boy crying. (b) That book on the table is mine. (c) From all sides rushed the howling pack of wolves.

III.—Parse fully the words in (a) and (b) above.

IV. Use the words: Around, snow, more, aim, and howl, as two or more different parts of speech.

V.—Correct, giving reasons for such corrections: (a) John walks slow. (b) He stole them pears. (c)

He was real sorry to hear it. (d) Each animal in the stable knows their own name.

VI.—Write: (a) A sentence containing an adverbial phrase. (b) A sentence containing an adverb modifying an adjective; and also one containing an adjective used as a noun.

VII.—(a) Give the relation of each word in the following sentence: The mob approached the prison and attempted to force open the gates.

FOURTH CLASS.

Candidates will do any six questions. Values—75 marks; 12½ marks for each question.

I.—State the different inflections of the pronoun, illustrating your answer by examples.

II.—Analyze fully: (a) Arriving upon the scene we found that we were too late. (b) Please to answer by return mail. (c) Have you seen the tall trees swaying when the blast is sounding shrill?

III.—Parse fully the italicized words in preceding question.

IV.—Use as different parts of speech five words from the sentence in Question II. State the part of speech of word used.

V.—What do you understand by Tense, Comparison, Intransitive Verb, Pronominal Adjective, and Analysis?

VI.—Correct the errors in the following sentences, giving reason for such corrections: (a) Seeing us approach we went away. (b) Let you and I go at once. (c) Wanted.—A woman to wash, iron and attend to three small children. (d) Whom did you say owns the house?

VII.—From house to house that wild scene amid rushed the devouring flames. State the relation of each word in the foregoing sentence.

ARITHMETIC—JUNIOR FOURTH.

Seven questions make a full paper. Values—15 marks each.

I.—Resolve into prime factors; 216, 289, 900, 1155, 13923.

II.—Find the greatest number that will divide 1397 and 2633, leaving the remainder 11 and 15 respectively.

III.—What is the capacity of the smallest cistern that can be filled in an exact number of minutes by either or both of two pipes, the first of which runs 48 gals. and the second 64 gals. per minute?

IV.—The difference in weight between two boxes of tea is 15 4-5 lbs., and the lighter box weighs 19¾ lbs. Find their total weight.

V.—A piece of cloth measured 27½ yds. before fulling, but only 25¾ yds. after fulling. How much did the cloth shrink?

VI.—Find the price to the nearest cent of 26½ bu. oats at 29¾ c. per bu., and 45¾ bu. wheat at 74 8-9c. per bu.

VII.—A man took 4678 steps in walking 2 227/1058 mi. What was the average length of his step?

VIII.—Find the value of 915 lbs. wheat at 64c. per bu., 425 lbs. oats at 32c. per bu., and 372 lbs. barley at 42c. per bu.

IX.—Divide 13½ lbs. tea into two parcels, one of which shall be 3¾ lbs. heavier than the other.

X.—Find the value of a pile of cordwood 6'-4" high, 39' long and 4' wide, at \$2.56 per cord.

ARITHMETIC—PART SECOND CLASS.

Juniors first seven questions; Seniors last seven. Five questions make a full paper.

I.—Find the sum of 7896 + 979 + 809 + 99 + 64537.

II.—John owns 97 acres of land, James 125 acres, Charlie owns 275 acres, and Robert owns 25 acres; how many acres do they own altogether?

III.—A man paid 275 dollars for a span of horses and 79 more than this for a carriage. For how much must he sell them both to gain 85 dollars?

IV.—Find the difference between 43763 and 4235.

V.—A man owing 769 dollars paid at one time 389 dollars, and at another 285 dollars. How much does he still owe?

VI.—Write in figures: XIX, XLIX, LXXXIX, XCIX, CIX.

VII.—Write in words: 101, 110, 805, 850, 1001.

VIII.—Find the result of: 7369 - 476 - 1945 + 3052 - 879.

IX.—The minuend is 1000, the subtrahend 799; find the remainder.

X.—A boy has 575 cents, he spends 75 cents for a book and 424 cents for a suit of clothes. How much had he left?