

acted upon may form a part. But to return to M. Poisson's proof, to which our attention was directed by finding it in Mr. Todhunter's book. It may sound a bold assertion to make concerning a proof published by such a man as Poisson, but we cannot help coming to the conclusion that it is a complete fallacy. We cannot give the proof at length, but the following general description of it will enable us to point out where the fallacy lies. Assuming that the direction of the resultant of two equal forces will bisect the angle between the directions of the two forces themselves, he takes two equal forces, P , inclined at an angle $2x$, whose resultant is R , and assumes $R = P f(x)$; his object being to determine the form of the function f . By resolving each of the forces P into two equal forces, Q , inclined at an angle $2z$; he arrives at the equation

$$f(x) \cdot f(z) = f(x+z) + f(x-z) \dots \dots (1)$$

This functional equation he has to solve, i.e., he has to find the most general solution, and to limit it by considerations derived from the special problem before him. This he proceeds to do as follows: "We see at once that $f(x) = 2 \cos cx$ is a solution, c being any constant quantity. We proceed to shew that this is the *only* solution, and that $c=1$." Mr. Todhunter, perhaps, scarcely conveys Poisson's meaning here. His words are: "Or je dis que cette expression de la fonction $f(x)$ est la seule qui satisfasse à l'équation (1), et que de plus dans la question qui nous occupe la constante c est l'unité."

As far as we can make out, the reasoning which follows is *not* intended to shew that the equation (1) admits of no other solution, (which we are required to take upon M. Poisson's assertion) but only that in the particular case before us $c = 1$. The steps by which it is endeavored to prove this are as follows. First, it is asserted that it is evidently true that $c = 1$, or that $f(x) = 2 \cos x$, when x is zero, for then the directions of the two forces P would coincide, and the resultant R would be $2P$, and we must therefore have $f(0) = 2$. Again he shews that the conditions of the problem are satisfied by assuming $f(x) = 2 \cos x$ in another particular case, viz., when $x = 60^\circ$ in which case the resultant $R = P$, which involves the assertion $f(60^\circ) = 1$ which as $\cos 60^\circ = \frac{1}{2}$ is satisfied by writing $f(x) = 2 \cos x$. A most ingenious proof is then inserted to shew that if the relation $f(x) = 2 \cos x$ is satisfied for $x = 0$ and for any other value of x , it must be satisfied for *all* values of x . The proof of this assertion is derived entirely from the equation (1) itself, and inasmuch as the object in view is altogether to choose among the different solutions of the equation that one which suits the physical