For the point n is the ultimate intersection of the intersections of the lines ab ... n and a'b' ... n meeting upon the curve line Bdef ... hikD', (prop. 2 and 3.) Now from the points A, describe through n the arc *an*, meeting the arc BD in the point *n*—it is evident that the distance An is equal to the distance A*n*, (Fig. 5.) Therefore the distance An will be equal to the determinate length of the circumference of the circle ABD, (Fig. 4.)

The intersection of the straight lines ab and a'b' in the point D, is independent of the nature of the curve line or arc BD'—for let BD', be any curve line described between the circular arc BD', and the curve line Bdef ... hikD, and describe the arcs BK' and BK, similar and equal to such curve line, it is evident that when the curve lines BK' and BK, come each to coincide with the similar and equal curve line BD'—the variations de'' and *ef''*—kk'' and *ii''*, must be the variations of the intercepted arc dk of the curve line BD'—and as the intersection n, of ab and ab', must be the ultimate intersection on d*k*, the point n consequently must be upon the intersection of the arc *nN*, (Fig. 5), and the curve line BD'; therefore this solution must be independent of the nature of the curve line BD'.

CASE SECOND.—*When the arc Bdef ... hikD', is the arc of a circle of the radius AC, described, through the points B and D'—intersecting either of the series of arcs 1K or 1''K only*, (*Lem.* 3, *Fig.* 5.)

SOLUTION FOURTH.

PROPOSITION 5.

THEOREM.

FIG. 8. Bisect the arc CD in the point D', and with the radius AC, describe through the points B and D' the arc BD', intersecting the arcs 1T, 2S, 3R, 4'Q, 3'O and 2'K, in the points d*ef* ... h*ik*. Then from the points C, with the distance C*e* and C*f* describe the arc *ee'* meeting the arc 1T, in the point *e'* and the arc 2S, in the point *f'*. Then from C as a center, with the distances Ck, Ci, Ch, &c., describe the arcs 2'K', 3'O', and 4'Q', &c.,—and from the same center with the distances Cd, C*e*, and C*f*, &c., describe the arcs 1''T', 2''S', and 3''R', &c.—Then through the points B and K', and B and K, describe the arcs BK' and BK, meeting the arc CD in the points K' and K; also from the point D' as a center with the distances D'*e'*, and D'*f'*, describe the arcs *e'e''* and *ff*; "the point *e''* shall be the variation of d towards *e''*, and *f''* the variation of *e* towards *f''*.

For in the same manner we have by (Fig. 7,)—The arc tu'' is the variation of t towards n, and nv'' is the variation of n towards v on the arc BK'—and t's'' is the variation of t' towards s, and sr'' is the variation of s towards r, on the arc BK. Now let the arc BK', move on the center B towards the arc BC, till it coincides with the arc BD'—the variation tu'' must coincide with the arc of variation d*e''*, and also the arc of variation nv'' must coincide with the arc of variation *ef''*; also let the arc BK, move on the center B towards the arc BD till it coincides with the arc BD'; the variation t's'' must coincide with d*e''*, and sr'' must coincide with *ef''*; hence the arcs de'' and *ef''*, are the variations on the common arc of intersections dk of the arc BD'.

PROPOSITION 6.

THEOREM.

FIG. 8. From the point D', with the distances D'*e''* and D'*f''*, and from the point B with the distances Bk, and B*i*, describe the intersections a and b; and through the points a and b, draw the straight line abn, meeting the circular arc BD' in the point n, and join A and n; the distance An, shall be the determinate length of the circumference of the circle ABD, (Fig. 4.)

For it has been demonstrated, (prop. 1,) that the point n, must be the point of ultimate intersection of the line ab, &c., or of all the intersections described through the series of points of variation *e''* and *f''*, &c., and through the points k and i, &c.—through which the curve line ab .. n, will coincide with its chord an, (Lem. 10.) Therefore the point n must also be on the intersection of the arc *n*N, (Lem. 3.), and the circular arc BD'; and An must be equal to A*n*, and equal to the circumference of the circle ABD, (Fig. 4.)