

of rivet bearing, and the movements of the structural elements upon each other. For such short members the strength depends very much upon the form of section and the method of fabrication.

(d) For compression members having a length intermediate between that of the very long column and the very short compression member, the slenderness ratio  $\frac{l}{r}$  does

affect the strength of the column. Just why this is true is not clear. It is evident that the reasoning about the effect of the length used in the derivation of Rankine's formula is not entirely tenable. It might even be thought that as the column of intermediate length does not deflect laterally to an extent which will explain failure, the strength of a column will remain nearly constant from the very short column to the vicinity of Euler's curve. However, the tests of columns show a gradual reduction in strength with increase in length, though the reduction is of a different character and generally less in amount than that given by Rankine's formula. In fact, tests show that a straight line inclined to the axis represents the results of tests of compression members of intermediate length very well—better than the Rankine formula. Failure in such columns may be by crinkling or wrinkling, the yield point being exceeded at places not necessarily at the centre of the length, and no special bowing or lateral deflection occurring in the amount which would be necessary to explain the failure, such as is seen in the failures of very long columns. In other words, failure seems to be due more to local causes than to general curvature along the whole length as is assumed in most column analyses. As has been indicated, reduction in strength in this intermediate field may be said to be directly proportional to the length of the column, though just why this is has not been determined.

(e) The extent to which length affects the strength, for columns of this intermediate length, seems to depend upon the shape of the section, the thickness of parts, and the method of making up the section, as whether the structural elements are riveted together, as in the case of built-up columns, or whether the piece is an integral rolled section. The tests of rolled H-sections made at Watertown arsenal (referred to in this discussion) may be expressed by the

$$\frac{P}{A} = 33,500 - 60 \frac{l}{r} \text{ for ultimate strength, up to the}$$

$$\text{limit of the tests, } \frac{P}{A} = 150, \text{ and of } \frac{P}{A} = 30,000 - 50 \frac{l}{r} \text{ for}$$

$$\text{the elastic limit of the columns. The tests of lap welded steel tubing may be expressed by the formula } \frac{P}{A} = 36,000$$

$$- 65 \frac{l}{r} \text{ for ultimate strength, for lengths up to the limit of}$$

$$\text{the tests, } \frac{P}{A} = 175, \text{ and for the elastic limit strength by}$$

$$\frac{P}{A} = 32,000 - 50 \frac{l}{r}. \text{ These equations for elastic limit}$$

strength are applicable for lengths up to the longest length tested. The equations for ultimate strength do not apply to

$$\text{tests of columns having a length of } \frac{l}{r} = 25 \text{ or less, for here}$$

there is insufficient length to complete the wrinkling action.

$$\text{Tests of the Gray type of column (with limits up to } \frac{l}{r} =$$

$$60) \text{ made at the University of Illinois are expressed by the equation } \frac{P}{A} = 36,500 - 155 \frac{l}{r}. \text{ It is evident that in this}$$

form of column, length has a much larger effect than in the two other forms. The general forms of column sections may be expected to have strength equations lying between this equation and the preceding two. An interesting fact brought out in the tests of the Gray columns (see Bulletin No. 56 of the Illinois Engineering Experiment Station) is that at small deformations the effect of length of column is very much less than at the higher deformations, and in this may be found a suggestion bearing upon the explanation of column action. It should also be noted here that in tests of built-up columns the load-deformation diagrams have a considerable curvature even at low loads, while in the tests of the rolled columns the low load-deformation diagram is nearly straight up to the yield point of the columns.

$$(f) \text{ The amount of the effect of the slenderness ratio } \frac{l}{r} \text{ through this range of intermediate lengths depends}$$

also upon the end condition of the column. The equations given in the preceding paragraph are for columns tested with flat ends which acted in the tests as fixed ends. The diagrams giving the results of tests of free-ended columns show a greater slope. Just what explanation best fits the existence of this greater slope, if general flexure is not the only cause of decrease of strength, cannot now be told. It must be understood, however, that end condition does affect the strength reduction for columns of intermediate length, and that a statement of the end condition assumed may well go with the formula.

It may be observed, too, that a single formula for all forms of sections and for different end conditions may not be expected to express the actual column strength.

Some very interesting questions have been raised in the discussion on compression members. One of these is, why

$$\text{not remove the limit of } \frac{l}{r} ? \text{ Why was this limit inserted}$$

$$\text{in the ordinance? Possibly because it was felt that there is too much uncertainty in the action of the column beyond}$$

$$\text{the length of } 120 \frac{l}{r}. \text{ Possibly it was done because it was}$$

appreciated that the end condition—whether round or fixed—so largely affects the strength of very long columns, and it was not expected that the brief statement in the ordinance could provide for the variations in condition likely to be found in practice. Besides, it has been thought possible by some that in a column formula for homogeneous columns the line which represents the strengths of columns of intermediate length may intersect Euler's curve instead of being tangent to it as commonly assumed.

Enough has been said to justify the remark that more experimental knowledge of column action is needed, for the variety of condition of section, of bearing, of length, and of connections. The view expressed by Mr. Horton, that a large number of laboratory tests are essential to give us an adequate understanding of column strength, has much to support it. The effect of form of section, of thinness of parts, of local crookedness, of end condition, of length and slenderness ratio for different end conditions and different forms of section, of form of lacing and its spacing, of lateral stiffness, of the effect of local overstress, of the effect of the rivet holes and of filling the holes with rivets, of the influence of slip of rivet—it would be easy to overrun Mr. Horton's 1,000 tests. As Mr. Horton says, "No mere mental