it is found that 24 equal taps opened together will empty it in 5½ min., and 15 of them will empty it in 13 min. How many will empty it in 33 min.?

11. If $\frac{a^3+b^3-c^3-d^3}{a-b+c-d} = \frac{a^2-b^2-c^2+d^2}{a+b+c+d}$

then will $\frac{ab-cd}{a-b+c-d} = \frac{ad-bc}{a-b-c+d}$

12. In what time will the discount on a sum of money be equal to the interest on the same sum for a year, money being worth 5 per cent. per annum?

13. Which of the following statements is more nearly correct?

$$\frac{10}{9.009} = 1.11 \text{ or } \frac{10}{1.11} = 9.009.$$

14. A square plot of ground by a river's edge containing one-tenth of an acre is inclined at 30° to the horizon. If the river rises until the plot is just submerged, find the pressure on the plot.

15. Solve the eqns.

$$\left(\frac{x}{y}\right)^{x+y} = \frac{y^4}{x}, \quad (xy)^{x+y} = y^4 x.$$

16. The population of a country would increase annually 5 per cent., were it not that emigration annually carries off ½ per cent. of the people; what will be the increase per cent. in the population after 5 years?

HONOR ALGEBRA,

JUNIOR MATRICULATION, 1879.

1. Define a fraction, and prove that

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Simplify

$$\frac{\frac{1}{1-a} - \frac{1}{1-b}}{\frac{1}{(1-a)b} - \frac{1}{(1-b)a}} \times \frac{\frac{1}{1-b} - \frac{1}{1-c}}{\frac{1}{(1-b)c} - \frac{1}{(1-c)b}} \times \frac{\frac{1}{1-c} - \frac{1}{1-a}}{\frac{1}{(1-c)a} - \frac{1}{(1-a)c}}.$$

Ans.—$a^2 b^2 c^2$

2. Describe methods of finding G. C. M. of two algebraical quantities.

Shew that $(a-b)(b-c)(c-a)$ is the G.C.M. of $(a+b)(a-b)^3 + (b+c)(b-c)^3 + (c+a)(c-a)^3$ and $(a-b)(a+b)^2 + (b-c)(b+c)^2 + (c-a)(c+a)^2$

We can arrange the first expression thus:—$(a^2-b^2)(a^2+b^2-2ab) + (b^2-c^2)(b^2+c^2-2bc) + (c^2-a^2)(c^2+a^2-2ac) = -2a^3b + 2ab^3 - 2b^3c + 2bc^3 - 2c^3a + 2a^3c = 2(a-b)(b-c)(c-a)(a+b+c)$.

And the second expression can be arranged thus:—

$(a^2-b^2)(a+b) + b^2-c^2)(b+c) + (c^2-a^2)(c+a) = -ab^2+ba^2-bc^2+cb^2-ca^2+ac^2 = -(a-b)(b-c)(c-a)$ ∴ $(a-b)(b-c)(c-a)$ is the G.C.M., and $2(a-b)(b-c)(c-a)(a+b+c)$ is the L. C. M.

3. Find the square root and the fourth root of $x+x^{-1}-4\sqrt{-1}(x^{\frac{1}{2}}-x^{-\frac{1}{2}}) - 6$.

If $x^4+2ax^3+bx^2+2cx+d$ is a complete square, prove that

$$a = \frac{c}{\sqrt{d}} = \frac{b-2\sqrt{d}}{a}.$$

The first expression can be arranged thus:

$$x + \frac{1}{x} - 2 - 4\sqrt{-1}(x^{\frac{1}{2}} - x^{-\frac{1}{2}}) +$$

$$(2\sqrt{-1})^2 = (x^{\frac{1}{2}} - x^{-\frac{1}{2}})^2 - 4\sqrt{-1}$$

$$(x^{\frac{1}{2}} - x^{-\frac{1}{2}}) - (2\sqrt{-1})^2 =$$

$$(x^{\frac{1}{2}} - x^{-\frac{1}{2}} - 2\sqrt{-1})^2 = (x^{\frac{1}{2}} - \sqrt{-1}x^{-\frac{1}{2}})^4$$

In the next expression if we extract the square root by the ordinary method, and, at the end of the operation, equate the coefs. of x and x^0 we shall have

$$\frac{b-a^2}{2} = \sqrt{d} \text{ and } a(b-a^2) = 2c$$

$$\therefore a = \frac{b-2\sqrt{d}}{a} = \frac{c}{\sqrt{d}}$$

4. Find the roots of the equation $ax^2 + bx + c = 0$.

What do the roots become when (1) $a = 0$, (2) $c = 0$; (3) $a = 0$ and $b = 0$?