GRAPHICAL METHOD FOR BEAM DEFLECTION

PRACTICAL SOLUTIONS TO COMPLICATED PROBLEMS IN THE DEFLECTION OF BEAMS, OBTAINED BY THE APPLICATION OF THE FOLLOWING GRAPHICAL METHOD.

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THE well-known formula for the deflection of a beam (due to the bending moment caused by the load), $\frac{d^2y}{dx^2} = \frac{M}{EI}$, where the values of x are horizontal and those of y vertical, M is the bending moment at any point, and E and I are the usual constants, affords a practical solution of such problems by a graphical method. Except for the simplest cases, the differential equation above is, at best, cumbersome to handle by the calculus, and, therefore, inefficient for practical purposes.

From this equation, $\frac{dy}{dx}$, which represents the slope of the elastic curve of the neutral axis at any point x, equals $\int \frac{M dx}{EI}$, and y, which represents the vertical deflection at any point x, equals $\iint \frac{M dx^2}{EI}$. Also observe that $M = \int S\,dx$ and that $y = \iiint \frac{S\,dx^3}{EI}$, where S is the shearing force at x. If E and I are constant the following graphical method will be found valuable for complicated cases:—

Given a 6-in., 12.25-lb. I-beam, with the span and loading shown in Fig. 1, E = 30,000,000 lbs. per sq. in. I = 21.8 inches^4. Required the maximum deflection.

The method is as follows: The shearing force curve was first plotted as shown in Fig. 1, and from this the bending moment curve was obtained as follows: Vertical strips 1 foot wide to scale were taken and the middle ordinate of each strip was laid off on a vertical line at the right-hand end of the beam. The point P was chosen as a convenient pole, and the curve constructed in the same manner as an equilibrium polygon. Note that the edges of the strips were connected by the strings and not the middle ordinates.

Again, taking the middle ordinates of the bending moment curve, in the same way a new curve, the slope curve, is constructed. The ordinates to this curve give the slope at any point. Here, it is to be noted that to draw these curves in their proper position they must be made to conform to certain known conditions. The value of the bending moment is zero at the end of the beam; therefore, its curve must pass through zero at that point. The slope of the deflection curve is zero at the middle of the beam; therefore the slope curve must pass through zero at that point; and it has been so drawn. Finally, from the slope curve the deflection curve has been drawn in the same way, passing through zero at the ends of the beam, because the deflection here is known to be zero from the conditions of the problem.

Care must be taken with the scales, but there is no difficulty involved if they are put down as follows:—

First Scale: Horizontal, 1 in. = 2 ft.; vertical, 1 in. = 500 lbs.; pole distance = 6 ft.

For the Second Scale multiply the pole distance into the vertical scale of the First Scale.

Vertical, 1 in. = 3,000 foot-pounds.
Pole distance = 6 feet.

In the Third Scale the vertical is obtained in the same way.

Vertical, 1 in. = 18,000 pound-feet^2.
Pole distance = 6 feet.

For the Fourth Scale, the vertical scale,
1 in. = 108,000 pound-feet^3.

The deflection at any point is now obtained by scaling the vertical ordinate at the point to the deflection curve in inches (use a scale graduated to 100ths of an inch). Multiply this by 108,000 ft.-pds. and by 12 to obtain the deflection in inches. The product EI in the denominator must also be inserted.

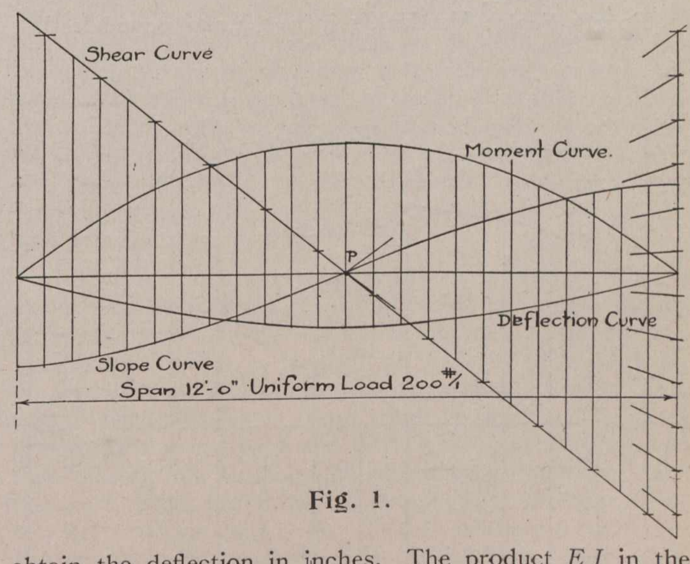

Fig. 1.

The results for this case are as follows:—

Scaled maximum ordinate on the deflection curve at centre = .50 inches.

$$\text{Deflection, } \frac{108{,}000. \times .50 \times 12^3}{30{,}000{,}000. \times 21.8} = .1427 \text{ inches.}$$

From the formula—

$$\text{Deflection, } \frac{5 \times 200. \times 12^4 \times 12^3}{384. \times 30{,}000{,}000. \times 21.8} = .1429 \text{ inches.}$$

Such extreme accuracy as this is due more to good fortune than to anything else, but it goes to show that the results so obtained are sufficiently accurate for practical purposes.

Let it be required to find the deflection of any point, "a," of the beam, with its loading, shown in Fig. 2. Fig. 3 shows the shearing force, bending moment, slope and deflection curves. The slope curve is not in its cor-