

struggle of ages, all that is due to mathematics, and we shall be centuries nearer the primitive barbarism of the race. Permit me to give in this connection Herbert Spencer's rapid review of the facts as to the worth of mathematics. For all the higher arts of construction, some acquaintance with mathematics is indispensable. The village carpenter, who, lacking rational instruction, lays out his work by Empirical rules learnt in his apprenticeship, equally with the builder of a Britannia Bridge, makes hourly reference to the laws of quantitative relations. The surveyor on whose survey the land is purchased; the architect in designing a mansion to be built on it, the builder in preparing his estimates; his foreman in laying out the foundations; the masons in cutting the stones; and the various artisans who put up the fittings; are all guided by geometrical truths. Railway making is regulated from beginning to end by mathematics; alike in the preparation of plans and sections; in staking out the line, in the measurement of cuttings and embankments; in the designing, estimating, and building bridges, culverts, viaducts, tunnels, stations. And similarly with the harbors, docks, piers, and various engineering and architectural works that fringe the coasts and overspread the face of the country; as well as the mines that run underneath it. Out of geometry too, as applied to astronomy, the art of navigation has grown, and so, by this science, has been made possible that enormous foreign commerce which supports a large part of our population, and supplies us with many necessaries and most of our luxuries. And now-a-days even the farmer, for the correct laying out of his drains, has recourse to the level—that is to geometrical principles. When from those divisions of mathematics which deal with *space* and *number*, some small smattering, of which is given in schools, we turn to that other division which deals with *force*, of which even a smattering is scarcely ever given, we meet with another large class of activities which this science presides over. On the application of rational mechanics depends the success of nearly all modern manufacture. The properties of the lever, the wheel and axle, &c., are involved in every machine—every machine is a solidified mechanical theorem; and to machinery in these times we owe nearly all production. Add to which that for the means of distribution over both land and sea, we are similarly indebted. And then let it be remembered that according as the principles of mechanics are well or ill used to these ends, comes success or failure—individual and national. The engineer who misapplies his formulæ for the strength of materials, builds a bridge that breaks down. The manufacturer whose apparatus is badly devised, cannot compete with another whose apparatus wastes less in friction and inertia. The ship-builder adhering to the old model, is out-sailed by one who builds on the mechanically justified wave-line principle. And as the ability of a nation to hold its own against other nations, depends on the skilled activity of its units, we see that on such knowledge may turn the national fate. Judge, then the worth of mathematics." Having determined beyond question what knowledge is of most worth—indeed actually essential to the progress of mankind—we have thereby determined what is of most value as a means of intellectual discipline; for "it would be utterly contrary to the beautiful economy of *nature* if one kind of culture were needed for the gaining of information, and another for the development of intellect. Everywhere throughout creation we find faculties developed through the performance of those functions which it is their office to perform; not through the performance of artificial exercises devised to fit them for these functions.

I shall be charged, perhaps, in consequence of these views with believing in the *absolute* sufficiency of Mathematics as a means of education, and with arrogance in rejecting the testimony of certain eminent men against the utility of Mathematical discipline. But I neither hold the all-sufficiency of Mathematics nor possess credulity enough to render a passive belief in any utterance, simply because it is sanctioned by illustrious names. There is a great deal of contradiction among the authorities cited against the value of Mathematics in education; and those whose evidence is not nullified by

mutual contradiction, are opposed by a greater number of more *credible*, because more *competent* witnesses. In fact the extravagance of the opposing testimony demonstrates its falsity. It is asserted that Mathematics are difficult only because they are too easy, that they determine thought to its feeblest development—that they actually dwarf the mental powers—that they contribute no advantage as a passport to Psychology or other sciences—that they lead to credulity and scepticism—and that a great genius cannot be a great mathematician. Descartes, Leibnitz, Newton, Euler, La Place, and a host of others—were they not men of genius? Or has the admiration of successive generations been only the tribute of a "blind credulity?" Then humanity has produced nothing great, and subjects for veneration must be sought among the forgotten—and soon to be forgotten—assailants of Mathematics. To show that I do not stand alone in my estimate of the value of Mathematics, it may be well to quote the evidence of a few of the witnesses who are *really competent* to give an opinion on the subject. *Kant*, after stating that the sure path of Metaphysical science has not yet been found, says: "It seems to me that the examples of Mathematics and Natural Philosophy are sufficiently remarkable to fix our attention on the essential circumstances which have proved so beneficial to them, and to induce us to imitate them so far as the analogy which, as rational sciences, they bear to Metaphysics may permit." *Cousin*, "the greatest philosopher of France," asserts the influence of Mathematics in the philosophy of *Kant*, and speaking of the Mathematic and idealistic character of the Pythagorean philosophy, says: "For Mathematics are founded on abstraction, and there is an intimate alliance between Mathematics and idealism; thence the Mathematical idealism that penetrates all parts of the Pythagorean system." And in reference to the Platonic philosophy, he observes:—"Abstraction is, therefore, the process, the instrument, of all good philosophy; this is also the process which characterizes the genus of *Plato*; hence, all that is true and sublime in the philosophy of *Plato*; hence his morality, his politics, and his *decided taste for Mathematics*; you perceive, in fact, that the Mathematical habit of considering, in quantities and dimensions, only their essential properties, was a happy preparation to Platonic abstraction." This is clear evidence of the power of Mathematics to develop abstraction, and prepare the mind for the accurate investigation of its own phenomena. As to credulity and scepticism, we have the testimony of the celebrated *Dr. Barrow*; "Mathematics deliver us from credulous simplicity, most strongly fortify us against the vanity of scepticism, effectually restrain us from rash presumption, and most easily incline us to due assent." *Lord Bacon* says:—"In the Mathematics I can report no deficiency, except it be that men do not sufficiently understand the excellent use of the pure Mathematics (he could have added *now* and the *still more excellent use of the applied Mathematics*), in that they do remedy and cure many defects in the wit and intellectual faculties; for if the wit be too dull they sharpen it; if too wandering they fix it; if too inherent in the sense they abstract it." And *John Stuart Mill* says: "The value of Mathematical instruction as a preparation for those more difficult investigations, society, government, etc., consists in the applicability, not of its doctrines, but of its method; the applications of Mathematics to the simpler branches of physics furnish the only school in which philosophers can effectually learn the most difficult and important portion of their art—the employment of the laws of simpler phenomena for explaining and predicting those of the more complex; these grounds are quite sufficient for deeming Mathematical training an indispensable basis of real scientific education, and one of the most essential qualifications for the higher branches of philosophy."

I think, then, that notwithstanding the dogmatic utterances of certain Metaphysicians who were almost totally ignorant of Mathematics, and the careless admissions of a few Mathematical Metaphysicians who sacrificed the certainty and stability of Mathematics for the aberrations of Psychology, enough has been adduced to establish the proposition that Mathematics are entitled to a high position as an INSTRUMENT OF EDUCATION.