(b) If $a^2+ab+b^2=\frac{a^3-b^3}{a-b}$, show without expanding that $(1+x+x^2)(1+x^3+x^6)(1+x^9+x^{18})(1+x^{27}+x^{54})=1+x+x^2+x^3+\ldots\ldots+x^{80}$.

The word "if" is out of place here, for $a^2+ab+b^2$ *is* the equivalent of $\frac{a^3-b^3}{a-b}$.

Then $1+x+x^2=1^2+1.x+x^2=\frac{1-x^3}{1-x}$

$1+x^3+x^6=1^2+1.x^3+(x^3)^2=\frac{1-x^9}{1-x^3}$

Similarly $1+x^9+x^{18}=\frac{1-x^{27}}{1-x^9}$ ... etc.

$$\therefore(1+x+x^2)(1+x^3+x^6)(1+x^9+x^{18})(1+x^{27}+x^{54})=\frac{1-x^3}{1-x}\cdot\frac{1-x^9}{1-x^3}\cdot\frac{1-x^{27}}{1-x^9}\cdot\frac{1-x^{81}}{1-x^{27}}=\frac{1-x^{81}}{1-x}=1+x+x^2+x^3+\ldots+x^{80}.$$

This question 2, is, in my opinion, much too difficult for the class in which it is set.

3. (a) This is book work and will be found in almost any work on algebra.

(b) Prove that if a and b be any two integers greater than unity, $a^3b-ab^3$ is always divisible by 3.

$$a^3b-ab^3=ab(a-b)(a+b).$$

If a or b is divisible by 3 the result follows. But if neither *a* nor *b* be a multiple of 3, they must be of the form $3m\pm1$ and $3n\pm1$. But, whichever sign of the ambiguity you take, either the sum or the difference of these is divisible by 3. ∴, etc.

4. (a) Solve $\frac{x+4a+b}{x+a+b}+\frac{4x+a+b}{x+a-b}=5$

This is $1+\frac{3a}{x+a+b}+\frac{4-3a-6b}{x+a-b}=5$.

$\therefore 3a(x+a-b)=3(a-2b)(x+a+b)$,

or $x\{a-a+2b\}=(a+b)(a-2b)-a(a-b)$

$\therefore x=-\frac{2b^2}{2b}=-b$.

(b) Solve $\frac{x-y}{a}=\frac{y-z}{b}=\frac{x+z}{c}=\frac{x-a-b}{a+b+c}$; assuming that,

if $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then $\frac{a+c+e}{b+d+f}=\frac{a}{b}$

Here, $\frac{x-y}{a}=\frac{x-y+y-z+x+z}{a+b+c}=\frac{x-a-b}{a+b+c}$,

Whence $2x=x-a-b$ $\therefore x=-(a+6)$

Then $x-y=a.\frac{x-a-b}{x+6+c}$

$\therefore y=x-a.\frac{x-a-b}{a+b+c}=-(a+b)-a.\frac{-2(a+b)}{a+b+c}$, by substituting for x, $=-(a+b).\frac{b+c-a}{b+c+a}$.

Otherwise, $\frac{y+z}{c-a}=\frac{x+z-(x-y)}{c-a}=\frac{-2(a+b)}{a+b+c}=\frac{y-z}{b}$.

$\therefore\frac{2y}{b+c-a}=\frac{-2(a+b)}{a+b+c}$, and $y=-(a+b)\frac{b+c-a}{b+c+a}$.

and $\frac{2z}{c-a-b}=\frac{-2(a+b)}{a+b+c}$, and $z=-(a+b)\frac{c-a-b}{c+a+b}$.