## ARTS DEPARTMENT.

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Our correspondents will please bear in mind, that the arranging of the matter for the printer is greatly facilitated when they kindly write out their contributions, intended for insertion, on one side of the paper ONLY, or so that each distinct answer or subject may admit of an easy separation from other matter without the necessity of having it re-written.

### SOLUTIONS,

to Problems in December number, by the proposer, J. L. Cox, B.A., Math. Master, Collegiate Institute, Collingwood.

183. Prove that $1+3n+\frac{3\cdot4}{1.2}\,\frac{n(n-1)}{\lfloor 2}+\frac{4\cdot5}{1.2}\,\frac{n(n-1)(n-2)}{\lfloor 3}+\&c.$

$=2^{n-3}(n^2+7n+8)$. (*Cam.*)

$$(x+1)^n=x^n+nx^{n-1}+\frac{n.n-1}{\lfloor 2}x^{n-2}+\frac{n(n-1)(n-2)}{\lfloor 2}x^{n-3}+\&c.$$

$$(1-x)^{-3}=1+3x+\frac{3\cdot4}{1.2}x^2+\frac{3\cdot4\cdot5}{\lfloor 3}x^3+\&c.$$

$$\therefore\ 1+3x+\frac{3\cdot4}{1.2}\,\frac{n(n-1)}{\lfloor 2}+\frac{3\cdot4\cdot5}{1.2.3}\,\frac{n(n-1)(n-2)}{\lfloor 3}$$

is the coefficient of $x^n$ in the product.

$$(1+x)^n(1-x)^{-3}=(1-x)^{-3}\{2-(1-x)\}^n$$
$$=2^n(1-x)^{-3}-x2^{n-1}(1-x)^{-2}+\frac{n(n-1)}{\lfloor 2}2^{n-2}(1-x)^{-1}$$

involving lower powers of $x$ than $x^n$; and picking out the coefficient of $x^n$ in the three we get $2^{n-3}(n^2+7n+8)$.

184.—Sum the series

$$\frac{1(x+2)}{2.3\ldots(x+1)}+\frac{2(x+3)}{3\cdot4\ldots(x+2)}+\frac{3(x+4)}{4\cdot5\ldots(x+3)}+\&c., \text{ ad inf.}$$

[NOTE.—This question was incorrectly given in the December number.]

$n^{th}$ term of given series

$$=\frac{n+(n+1+x)}{(n+1)(n+2)\ldots(n+x)}=\frac{(n+1)(n+x)-x}{(n+1)\ldots(n+x)}$$
$$=\frac{1}{(n+2)\ldots(n+x-1)}-\frac{x}{(n+1)\ldots(n+x)};$$
$$\therefore\ S_n=C-\frac{1}{(n+3)\ldots(n+x-1)(x-3)}+\frac{x}{(n+2)\ldots(n+x)(x-1)}.$$

Putting $n=0$,

$$C=\frac{2}{\lfloor x-1\,(x-3)}-\frac{x}{\lfloor x\,(x-1)};$$
$$\therefore\ S_n=\frac{(x+1)}{(x-1)(x-3)\lfloor x-1}-\frac{1}{(n+3)\ldots(x-3)}+\frac{x}{(n+2)\ldots(n+r)(x-1)};$$
$$\therefore\ \text{sum to inf.}=\frac{x+1}{(x-1)(x-3)\lfloor x-1}.$$

185. Find the sum of $n$ terms of the series

$$\frac{1}{(x+2y+3z)(2x+3y+4z)}+\frac{1}{(2x+3y+4z)(3x+4y+5z)}+\ldots$$

The $n^{th}$ term of the above is

$$\frac{1}{\{nx+(n+1)y+(n+2)z\}\{(n+1)x+(n+2)y+(n+3)z\}}$$
$$\therefore\ S_n=C-\frac{1}{\{(n+1)x+(n+2)y+(n+3)z\}\{(x+y+z)\}}$$
$$O=C-\frac{1}{(x+2y+3z)(x+y+z)}$$

and $C=\frac{1}{(x+y+z)(x+2y+3z)}$;