If the sewer is constructed in soft or compressible soil, the whole section, including the invert, should be treated as an elastic bent beam and the loading must include an upward pressure on the invert equal to the total vertical load.

A number of methods have been published for the analysis of the elastic arch. Of these the simplest is that presented in Green's "Trusses and Arches." Professor Green worked out bending moments in the parabolic arch for unit loads. He also presented constants for the semi-circular arch. Green's constants for the semi-circular arch have been extended by Mr. A. E. Lindau (Trans. Am. Soc. C.E., Volume 51), and put into the same tabular form as was originally given for the parabolic arch. Green's analysis is based on a constant ring thickness. It is not correct for the usual case in which the arch increases in thickness from crown to springing line and some idea of the error involved is given in Lindau's paper. Although inaccurate, this method is very convenient, in that by its use we are able to calculate moments directly from the loading without the previous assumption of an arch thickness. For the smaller sewers and not for all purposes and where the variation in ring thickness is not great, it is sufficiently accurate.

In the writer's practice, this method has been developed into a set of formulas applicable to the semi-circular arch. These formulas give the moment at each 10-degree point in terms of the mean radius of the arch and of the depth of fill over the crown. In the more important work, these formulas are used in order to determine approximate dimensions for an arch which is later to be analyzed by one of the more accurate methods. As the accurate methods must be applicable to all shapes of arches and variations in thickness, it is impossible to reduce them to any very simple form.

In detailing the arch from the calculated bending moments, it will usually be found advisable to use two full sets of reinforcement, that is, on the inner and outer face. If it were known positively that reverse moments could never occur, for example, if it were impossible in the case of a semi-circular arch that the horizontal force could predominate, it would be reasonable to omit a portion of one set of reinforcement or possibly to cross one set over from the inner to the outer face, but this generally cannot be insured and the full reinforcement should be put in even if only as an added factor of safety and for the sake of standardization. Another reason why the arch cannot be designed too closely is that any particular section, if multiplicity of sections is to be avoided, must be designed for variations of loading over a considerable range.

In the St. Louis work, where a considerable length of one size sewer and fairly constant soil conditions occur, it has been the practice to design a section for each 5 feet in depth of loading and to detail these sections for soft ground foundation, for hard bottom and for deep rock cut. A designer cannot follow too closely the calculated thickness of the arch, as some consideration must be given to the shape of the outside as well as the inside of the sewer. For example, if the sewer is to be built in a trench with vertical sides, it would be found much simpler to make the outside of the sewer vertical to some point above the spring point of the arch rather than to carry a small batter all the way down to the bottom of the sewer. This is because of the fact that it would cost less to fill in the small wedge-shaped space with concrete than to attempt to place and remove outside forms in the limited space available.

There seems to be no uniformity in practice as to the longitudinal reinforcement. A certain amount of steel is usually required in this direction to properly tie in the transverse bars and ½ or ¾-inch bars are often used on about 2-foot sections in both faces. If the sewer is to be constructed in hot weather and particularly in shallow cut, it might be advisable to increase the amount of longitudinal steel in order to distribute shrinkage cracks, but under other conditions this seems hardly necessary as the range of temperatures in the completed sewer is very small, probably varying from about 40 degrees F. in winter to 70 degrees F. in summer, unless steam or hot wastes are permitted to enter.

## RELATION OF SPECIFIC GRAVITY TO DEGREES BAUME.

The use of the Baumé scale is a frequent source of annoyance as well as convenience to those engaged in industries in which it is used, more especially in its connection with petroleum products for use in roadwark. The relation between the two scales has been aptly explained by H. W. Bell in "Western Engineering," who says that a natural and common mistake due to the use of Baumé scale is in the mixing of different gravities of oil to obtain a product of certain gravity on the assumption that the Baumé scale is a direct measure of specific gravity. To illustrate: We have 10,000 bbl. of 16° Baumé oil and wish to know the amount of 25° Baumé oil necessary to add to bring the product up to 20° Baumé. The method of solving by proportion, without converting Baumé to specific gravity easily suggests itself, and the result would be as follows: $10{,}000\,(16) + 25A = 20\,(A + 10{,}000)$, and $A = 8{,}000$, the apparent amount to be added. But Baumé gravity is not a direct measure of specific gravity, and specific gravity $= \dfrac{140}{130 + \text{Baumé gravity}}$. The general form of equation would be $A_s G_s + A_a G_a = G_x (A_s + A_a)$, where $A_s$ is amount of oil at the start; $A_a$ is amount to be added; $G_s$ is specific gravity of $A_s$; $G_a$ is specific gravity of $A_a$; and $G_x$ is specific gravity of mixture. Further, let $B_s$, $B_a$ and $B_x$ be the Baumé gravities of the oil to start, oil added, and the mixture, respectively; and putting in terms of Baumé,

$$\frac{140\,A_s}{130 + B_s} + \frac{140\,A_a}{130 + B_a} = \frac{140\,(A_s + A_a)}{130 + B_x}$$

$$\text{Solving, } A_a = \frac{A_s\,(B_x - B_s)\quad(130 + B_a)}{(B_a - B_x)\quad(130 + B_s)}$$

Using this formula and solving the given problem, the amount to be added is 8,493 instead of 8,000 bbl. If only 8,000 were added the theoretical gravity would be 19.86°, and it might easily result in refusal of purchaser to accept the shipment as 20° oil.

In the consideration of the effect of water content upon gravities, we know that they are lowered unless the gravity of the pure oil is 10° or less. To show the effect of water on specific gravity of oil mixture we can write the equation $G_o\,(100 - P) + P = 100\,G_m$ and $G_o = \dfrac{100\,G_m - P}{100 - P}$, where $G_o$ is specific gravity of pure oil, $G_m$ is specific gravity of mixture, and P is the percentage of water. Substituting Baumé degrees for specific gravity and letting $B_o$ and $B_m$ equal the Baumé gravities of pure oil and the mixture, respectively, the result is

$$\frac{140}{130 + B_o} = \frac{\dfrac{14{,}000}{130 + B_m} - P}{100 - P}$$

$$\text{Solving, } B_o = \frac{14{,}000\,B_m - 10\,P\,(130 + B_m)}{14{,}000 - P\,(130 + B_m)}$$