

Mathematical Department.

FIRST-CLASS ALGEBRA PAPER,

Set for Pupils' Examinations of the College of Preceptors,
Christmas, 1883.

1. Given $4x-5y=18=3(5x+2y)$, and $z=0$, find the value of $\sqrt{(3x+y-2z)(5x-4y+2x)-7z^2+2x^2-3y^2-5xy}$.

2. Prove, in any way, that
 $a^2(b-c)(b+c-a)^2+b^2(c-a)(c+a-b)^2+c^2(a-b)(a+b-c)^2$
 $=(a-b)(b-c)(c-a)[2(ab+bc+ca)-a^2-b^2-c^2]$.

3. Reduce to its simplest form

$$\frac{4x^2-3x+5}{x^2+1} - \frac{1-2x}{x^2+x+1} + \frac{6}{-x}$$

4. Solve the simple equations:—

$$(i.) 1 - \frac{1-\frac{3x}{5}}{2x} + \frac{1+\frac{3x}{2}}{5x} = \frac{2+5x-x^2}{10+21x-10x^2}$$

$$(ii.) (2x-3)(3y+5)-2(x-y)=(4\cdot5-3x)(3\cdot5-2y)+12\cdot5 \quad \left. \begin{array}{l} 3x=2y+3\cdot75 \end{array} \right\}$$

5. Solve the quadratic equation:

$$\frac{7x-2}{9} - \frac{3x+4}{4-3x} = \frac{9x+1}{7}$$

6. A Bill upon which 576 Members voted was lost on a division; subsequently, the same Members voting, it was carried by a majority half as large again as it was originally lost by, and the majority in the latter case equalled the number of those who first voted for the Bill. Find how many voted on each side, and how many Members changed their minds.

7. A messenger has to go a journey of 20 miles, and times himself to do it in a certain time. After walking 8 miles, he is delayed one hour, and is consequently obliged to increase his former rate by two miles an hour. Find his original rate of walking.

8. What values of p and q will make $4x^4+4px^3+16x^2+qx+9$ a perfect square?

SOLUTIONS.

1. $\left. \begin{array}{l} 4x-5y=18 \\ 5x+2y=6 \end{array} \right\}$, therefore $x=2$, $y=-2$, and $z=0$, therefore

$$\text{Expn.} = \sqrt{(6-2)(8+4)+8-12+20} = \sqrt{(48+16)} = \sqrt{64} = 8.$$

2. If $a=b$ the left hand member vanishes. $\therefore a-b$ is a factor, and hence $(a-b)(b-c)(c-a)$ is a factor, for the expression is symmetrical with respect to a , b , and c . It is of five dimensions, and we have found 3 factors, hence there must be only another factor of two dimensions of the form $P(a^2+b^2+c^2)+Q(ab+bc+ca)$. Hence put left hand member

$$=(a-b)(b-c)(c-a)[P(a^2+b^2+c^2)+Q(ab+bc+ca)]$$

where P and Q include all numerical factors. Then to find P and Q put $c=0$ on both sides and we have

$$a^2b(b-a)^2+b^2a(a-b)^2+0=(a-b)(-ab)[P(a^2+b^2)+Q(ab)]$$

or, dividing through by $-ab(a-b)$, this becomes

$$-(a^2+b^2)+2ab=P(a^2+b^2)+Q(ab),$$

from which it is plain that $P=1$, and $Q=2$, and consequently left hand member

$$=(a-b)(b-c)(c-a)[-a^2-b^2-c^2+2(ab+bc+ca)].$$

See May Number—Elementary Algebra.

3. Expression $= (4x^2-3x+5+2x^2-3x+1-6x^2-6x-6)+(x^2-1)$
 $= -12x \div (x^2-1) = 12x \div (1-x^2).$

4. (i.) Sinister member

$$= 1 - \frac{5-3x}{5-2x} + \frac{2+3x}{2+5x} = \frac{x}{5-2x} + \frac{2+3x}{2+5x} = \frac{10+13x-x^2}{10+21x-10x^2}$$

Therefore, since denominators are equal, we have

$$2+5x-x^2=10+13x-x^2 \text{ and } 8x=-8, \therefore x=-1.$$

(ii.) $6xy+10x-9y-15-2x+2y=15\cdot75+10\cdot5x-9y+6xy+12\cdot5$,
 $\therefore 18\cdot5x+2y=43\cdot25$
 $\left. \begin{array}{l} 3x-2y=3\cdot75 \end{array} \right\}, \therefore 18\cdot8x=47, \text{ and } x=\frac{47}{18\cdot8}=\frac{1}{2}=2\cdot5.$

Hence $2y=75-3\cdot75=-3, \therefore y=-1\cdot5.$

$$5. \quad \frac{3x+4}{4-3x} = \frac{7x-2}{9} = \frac{9x+1}{7} = \frac{32x+23}{63},$$

$$180x+252=96x^2-59x-92, 96x^2-248x-344=0,$$

$$\text{i.e., } 12x^2-31x-43=0=(x+1)(12x-43),$$

therefore $x=-1$ or $3\frac{1}{12}.$

6. Let x = No. who voted for (first division), then $576-x$ = No. against, therefore first majority $=576-2x$. Hence, taking account of second division, we have

$$x=\frac{1}{3}(576-2x) \text{ and } x=216.$$

Hence, in first division, 216 voted for, and 360 against. The majority in second division $=216$,

$$\text{therefore voters against} = \frac{1}{2}(576-216) = 180,$$

$$\text{and voters for} = 396.$$

Hence 180 changed their minds.

7. Let x = original rate, \therefore time originally allowed $\frac{20}{x}$.

$$\text{hence } \frac{20}{x} = \frac{8}{x} + 1 + \frac{12}{x+2}, \text{ i.e., } 1 = \frac{1}{x} - \frac{12}{x+2} = \frac{24}{x(x+2)}$$

$$\therefore x^2+2x-24=0, \text{ and } x=4 \text{ miles per hour.}$$

$$8. \quad \frac{4x^4+4px^3+16x^2+qx+9}{4x^4} = \frac{4px^3+16x^2}{4x^4} + \frac{qx+9}{4x^4}$$

$$\begin{array}{r} 4x^2+px \quad \left| \begin{array}{l} 4px^3+16x^2 \\ 4px^3+p^2x^2 \end{array} \right. \\ \hline (16-p^2)x^2+qx+9 \\ 4x^2+2px+3 \quad \left| \begin{array}{l} 12x^2+6px+9 \end{array} \right. \end{array}$$

By conditions of question, $16-p^2=12$, $q=6p$,

$$\therefore p^2=4 \text{ and } p=\pm 2, \therefore q=\pm 12.$$

Or we may solve the question by writing

$$4x^4+4px^3+16x^2+qx+9=(2x^2+ax+3)^2,$$

and then equate the coefficients of like powers on the two sides of the equation. The result is the same.

THE EXAMINATION OF STUDENTS IN TRAINING
COLLEGES (ENG.), DECEMBER, 1883.

ARITHMETIC.

1. Find the length of the edge of a cubical tank that contains 2,143 gallons and 3 quarts of water. Given, that 1 gallon of water weighs 10 pounds, and 1 cubic foot of water weighs 62½ pounds.

2. A bicyclist ran $4\frac{1}{2}$ miles in 17 minutes. The distance made in the last minute was $\frac{1}{2}$ of that made in the first minute; and the distance in each successive minute was less than that made in the preceding minute by the same quantity. Find the average and the decrease per minute.

3. A money lender charges 10% on a three months' bill, taking his 10% from the amount of the loan when the money is lent. What yearly income will he have from a capital of £3,561 constantly employed in this way?

4. If cloth 42 inches wide, weighing $6\frac{1}{2}$ ounces per yard, and made of wool worth 3s. 9d. per pound, be sold for 8s. 2d. per yard, how many ounces will there be in a yard of cloth 27 inches wide, made of wool worth 6s. 5d. per pound, and selling at 5s. 6d. per yard?

5. If an income tax of one penny in the pound produce £3,857, 216 lls. 1½d., and the capital of the country be supposed to produce an income of 2½%; find the capital of the country.

6. A put into a partnership £4,815 for one year and five months; B put in £3,531 for two years and one month; C put in £6,099 for ten months. Their profit was £1,926. What was the share of profit of each?

7. A bill for £3,511 10s. 6½d. was discounted for £3,497 3s. 1½d. at the rate of 2½%, simple interest per annum. For what time was it drawn?