Mathematics.

All communications intended for this column should be sent before the 20th of each month to C. Clarkson, B.A., Seaforth, Ont.

CORRESPONDENCE.

THE editor presents his New Year's compliments to the legion of correspondents who have generously snowed him up during the past month under an avalanche of queries and answers. What charm resides in these dry and prosy old problems? Their interest and vitality seem to remain in spite of all counter attractions. Probably the sense of power gives a delight similar to that produced by healthy, vigorous exercise in the gymnasium. As mind-sharpners and thought-correctors, mathematical studies do not seem likely to lose in the twentieth century their present imperial position in every great system of education. For training the working faculty, for securing independence, accuracy, and rapidity of thought, and for producing an abiding confidence in the unseen and immaterial, mathematics is undoubtedly the finest educational instrument yet invented. But mathematicians should beware of one-sided development, no matter how vigorous it may be. The mathematical all-in-all is apt to be a very ignorant and extremely dogmatic creature. Biography, history, poetry, eloquence, biology, chemistry, philology, ethnology, logic, psychology,—all these have easier subject matter than mathematics, and present unlimited fields of recreation and delight into which every prudent mathematician will make long excursions.

On the other hand, our studies have some beauties that are often overlooked by outsiders. If the rhetorician defines eloquence as the art of compelling the hearer to accept the conclusions of the speaker, what orator is more eloquent than rare old Euclid? If imagination is one of the loftiest characteristics of poetry, why should not trigonometry be classed as poetic composition, seeing that the interpretation of such expressions as $(\cos\theta + \sqrt{-1}\sin\theta)r$ requires greater effort of imagination than would suffice to interpret any ten lines in Milton or Shakespeare? Kingdoms may rise and fall, but the domain of pure mathematics will abide undestroyed and indestructible after the whole world has turned to coal; and therefore when new worlds spring into existence we hope our army of contributors may be ready to snow up the editor of some other mathematical column, with another avalanche of problems.

In the meantime we are glad to welcome a bright young book into our midst with the aroma of the present century about it. We recommand all our friends to become acquainted with the newly authorized text-book, "McKay's Elements." Modern methods have been judiciously introduced, proper gradation has been looked after, difficulties have been reserved to the appropriate stages, old prejudices have been buried out of sight, rote work has been effectually discouraged,—this is the high praise deserved by the new text book.

J.N.F., Colborne, argues that No. 25, solved in the second column of page 217 December number, does not admit of solution, because the words, "b hours behind time," should mean that the train or person ought to have passed the given point b hours before it actually did; and as we do not know what part of the journey has already been completed in losing the b hours, we can tell nothing about the remainder of the trip. If he will substitute numbers for a, b and c, we think he will probably reverse his judgment.

J.N.F., and Miss J.C.G., Ingersoll, both believe that the bankrupt mentioned by B.B. Formosa, on page 217, "was evidently a swindler." The law of libel is rather strict, and we warn all our correspondents to have a care. Mr. Thomas F. Flaherty, of Lucan, effectively defends the unfortunate man as follows:—

He loses $\frac{1}{5}$ of \$20,000 = \$4,000 on part of his nominal assets.

Next he actually collects ($\frac{4}{5}$ his liabilities – \$4000)

Then he loses $\frac{1}{25}$ of what he has collected, *i.e.* 4%; ∴ on the whole he realizes only $\frac{24}{25}$ ($\frac{4}{5}$ liabilities – \$4000, and this amounts to 60% of all his liabilities;

∴ ($\frac{4}{5}$ liabilities – \$4000) $\frac{24}{25} = \frac{3}{5}$ liabilities;

or $\frac{32}{25}$ liabilities – 6400 = liabilities;

∴ $\frac{7}{25}$ liabilities = 6400; and liabilities = \$22,857$_7$.

The same thing may be shown algebraically:

Let 4k = assets, then 5k = liabilities, ∴ $\frac{24}{25}$(4,000) = 3k

∴ 7k = 32000, and 5k = 22857$\frac{1}{7}$, 4k = 18285$\frac{5}{7}$. k –

Problem b, p. 217, Dec 1st. J.N.F. rightly says that this problem can be solved only approximately without using the calculus for determining the loss for "edgings." A good practical solution is given by Miss J.C.G., by squaring the log first and then sawing it into inch lumber: Thus side of square = $\sqrt{(\frac{1}{2} \text{ of } 27^2)}$ = 19.09 inches, ∴ make 14 cuts with the saw, taking out 5¼ inches, which leaves 13.84 inches for the 15 boards, and each board .922 inch thick. Then the amount of lumber = 15 × 14 × 19.09 ÷ 12 = 334¼ nearly. Mr. Flaherty makes it 378. If the log were 28½ inches in diameter we might make 20 cuts taking out 7½ inches in sawdust and giving 21 boards that would still require edging. The exact solution can scarcely be given without employing the higher analysis, which would interest only a small minority of our readers. If the log is supposed to taper the problem becomes more difficult.

Mr. S. Anderson, Walton, gives the following solution of No. 5, II., page 239, in H. Smith's Arithmetic:

First cask has 12 wine + 18 water

Second cask has 9 wine + 3 water

Both casks have 21 wine + 21 water. Hence mix the two casks and draw off 14 gals. of the mixture, and we shall get 7 wine and 7 water as required. Solved also by numerous contributors, using fractions, ratios, &c. Perhaps it would accord better with the exact requirements of the problem to take one-third of each cask and mix them together: (4 wine + 6 water) + (3 wine + 1 water) = 7 wine + 7 water.

Mr. S.A. also gives the solution to No. 5, IV., p. 199, H. S. Arithmetic:

500 bbs. at \$7 = \$3,500; 4% com. off leaves \$3,360 to be divided. Let 100 represent the quality of B's flour, ∴ 100:110:116, *i.e.* 1:$\frac{11}{10}$:$\frac{29}{25}$ will represent the flour reduced to B's standard. Hence 156.137½:261 represents the total amounts at B's standard.

Therefore give A, B, C, 275, 300, and 522; 1097th of the money respectively, *i.e.*, \$842.30, \$918.87, and \$1598.83. Solved by a large number of correspondents.

The reference No. 143, p. 247, H. S. Arithmetic, may have been meant for No. 143, p. 274. Mr. Thos. Cottingham, Rye, Ont., does it thus:

$\frac{1}{4}$ and $\frac{1}{5}$ are respectively emptied per hour.

Hence in a certain No. of hrs. the first has lost $\frac{1}{4}$ × No.

And the second has lost $\frac{1}{5}$ × No.

And there are left 1 – $\frac{1}{4}$ x No.; and 1 – $\frac{2}{5}$ × No.

Therefore 1 – $\frac{1}{5}$ No. = 2 (1 – $\frac{1}{4}$ No.)

Or 10 – 2 No. = 20 – 5 No.

∴ 3 No. = 10, No. of hrs. = 3⅓.

Solved algebraically by numerous contributors. We may avoid fractions by supposing each cask to contain 20 gals., then the first loses 5 gals. per hour, and the second 4 gals. per hour

∴ (20 gals. – 5 gals. × No. hrs.) 2 = 20 gals. – 4 gals. × No. hrs;

Or 40 gals. – 10 gals. × No. hrs. = 20 gals. – 4 gals. × No. hrs.

But 10 gals. × No. hrs. = 10 gals. × No. hrs.

∴ 40 gals. = 20 gals. + 6 gals. × No. hrs.

Or 20 gals. = 6 gals. × No. hrs.

∴ 20 gals. ÷ 6 gals. = No. hrs. = 3⅓ hrs.

T. C. asks for a solution of this problem:—"A field contains 300 acres, and is 3½ times as long as it is broad. Find the perimeter." That will depend on the angles contained by the sides. If it is rectangular,

Length by breadth = 300 ac.

Or 3½ breadth × breadth = 300 × 160 sq. rods.

∴ $(\text{Breadth})^2 = 300 \times 160 \times 2 \div 7$

∴ Breadth $= \sqrt{(100 \times 16 \times 60 \div 7)}$

$= 10 \times 4 \times 2\sqrt{(15 \div 7)} = \frac{80}{7}\sqrt{105}$

∴ length $= \frac{7}{2} \times \frac{80}{7}\sqrt{105} = 40\sqrt{105}$

∴ perimeter $= \frac{720}{7}\sqrt{105}$ rods $= 720 \times 10.247 \div 7$ nearly.

$= 720 \times 1.46385+ =$ etc.

We add the following solution which will probably interest a good many of our readers:

A and B put in \$3,400 into business; A's money was in 12 months, and B's 16 months. On settlement A received \$2,070 as his share, and B \$1,920. What capital did each invest?

Solution:—Let $S_1 G_1$ be A's stock and gain

Let $S_2\ G_2$ be B's stock and gain

Then $S_1 + S_2 = 3{,}400$; $G_1 + G_2 = 590$

$S_1 + G_1 = 2{,}070$; $S_2 + G_2 = 1{,}920$.

Also, the gains are proportional to the capital of each for one month.

$$\therefore \frac{3S_1}{4S_2} = \frac{G_1}{G_2} \qquad \therefore \frac{3S_1 + 4S_2}{4S_2} = \frac{G_1 + G_2}{G_2}$$

$$\text{Or, } \frac{3(S_1 + S_2) + S_2}{4S_2} = \frac{G_1 + G_2}{G_2}$$

$$\therefore \frac{10200 + S_2}{4S_2} = \frac{590}{G_2}; \text{ but } S_2 = 1920 - G_2$$

$$\therefore \frac{10200 + 1920 - G_2}{4(1920 - G_2)} = \frac{590}{G_2} = \frac{12120 - G_2}{7680 - 4G_2}$$

∴ $590 \times 7680 - 2360\,G_2 = 12120\,G_2 - G_2{}^2$; or using x for G_2

$x_2 - 14480\,x + 590 \times 7680 = 0$

i. e. $x_2 - x(14160 + 320) + 14160 \times 320 = 0$

Or $(x - 320)(x - 14160) = 0$

Or $x = 320$, or $14160 = G_2$

∴ 270, or $-13570 = G_1$

∴ $S_1 = 2070 - G_1 = 1800$, or 15640

$S_2 = 1920 - G_2 = 1600$, or -12240.

Thus the positive solution, \$1,800 and \$1,600, is the only one that applies to the problem in its ordinary interpretation.

The negative solution represents an imaginary partner ship somewhat like this: A, with a capital of \$15 640 takes his son B, who is in debt to the extent of \$12,240 into business with him. The firm gains \$590 and pays off B's debts. The following figures will make this clear:

A's capital =	\$15,640 cash	A's loss........ =	\$13,57
B's " =	12.240 debt	B's gain........ =	14,16
Cash capital	\$ 3,400	Total gain...... =	\$ 590
A's capital........	\$15,640	B's debts.........	\$12,240
A's loss...........	13,570	B's gain..........	14,160
A withdraws......	\$ 2,070	B withdraws......	\$ 1,920

So that the father pays the son's debts and gives him \$1,920 to start him in business again.

NOTE.—In writing to this column always send the *question itself* as well as the reference. If the problem is not likely to be of *general* interest, or if a prompt reply with full explanations is desired, write privately to the editor who gives instruction by correspondence to those who are unable to attend high schools or colleges.

HOME-MADE PICTURES.

WHAT the carpenter's tools are to him, or the house-keeper's utensils to her, the home-made charts, number cards, material for busy work and collections of curiosities become to the teacher who aims to work skilfully and systematically.

A system of charts become an absolute necessity in most schools, since blackboard room is limited and the record of the work on any subject must be erased to make room for another.

Light manilla paper is inexpensive, and cut into sheets of suitable size may be formed into very good charts.

I have one formed for Language. I cut up one of McLaughlin Bros.' publications, "Domestic Animals," and secured, at a cost of twenty-five cents, very good colored pictures of a cow, a calf, a horse, a sheep, a dog, and a family of cats.

These serve for topics for Animal Lessons as well as Language; from time to time I have added bright colored pictures that delight the children and serve as aids to Language or Reading Lessons.

The children are never weary of using them and do very satisfactory work with them.—*The School Teacher.*

MATERIAL AIDS IN SCHOOL GOVERNMENT.

FIRST among these is a *good classification* of the school. The scholar who is so classed and directed in his studies that he is encouraged to study the right thing at the right time, and by the right method, will, as a rule, cause but little disorder.

Second.—A clean and well-arranged room has a very wholesome influence upon the order of the school. Cleanliness and order beget a spirit of respectfulness, while a suitable arrangement of the desks and other furniture helps to regulate the posture and movements of the scholars.

Third.—A wholesome industry among the pupils. Find a teacher who maintains a spirit of earnest work in his school, and you find one who is regarded as a successful governor. "The best order," says Calkin, "does not consist in maintaining any fixed posture, nor in absolute quietness, but rather in that interested attention to the lessons which so occupies the minds of the pupils as to leave no inclination for disorder."

Fourth.—The *comfort* of the *pupils* as secured by a proper *temperature* and *ventilation* of the schoolroom. The experienced teacher soon learns that the exercise of proper precaution in looking after the temperature and ventilation of the room will always yield a rich reward in the order and good spirit of the school.—*John W. Woody, in The Student.*

OLD London is not the only place where starving children are swept in crowds into the public schools. Appalling disclosures recently resulted from an inquiry set on foot in Vienna. Upward of 4,000 children were suffering the pangs of hunger, some of them being on the verge of starvation and not a few of the unfortunate little ones had died. Active measures for relief were adopted by the charitably disposed.