UNIVERSITY WORK.

MATHEMATICS.

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SELECTED QUESTIONS.

By Wilbur Grant, Collegiate Inst., Toronto.

6. Show that

$$u^3+x^3+y^3+z^3=(u-x)^3+(u-y)^3+(u-z)^3+6xyz,$$

if $u=x+y+z$

$$(u-x)^3+(u-y)^3+(u-z)^3+6xyz$$
$$=(u-x)^3+(u-y)^3+(x+y)^3+6xy(u-x-y)$$
$$=u^3-3u^2x+3ux^2-x^3+u^3-3u^2y+3uy^2$$
$$-y^3+x^3+3x^2y+3xy^2+y^3+6uxy$$
$$-6x^2y-6xy^2$$
$$=u^3+\{u^3-3u^2(x+y)+3u(x+y)^2-(x+y)^3\}+x^3+y^3$$
$$=u^3+x^3+y^3+z^3.$$

7. Solve the equations

(*a*) $\dfrac{x-a}{x-b}+\dfrac{x-b}{x-a}+2=0.$

(*b*) $x+y=a^2\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$

$\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{b}+\dfrac{b}{xy}.$

(*a*) $\dfrac{x-a}{x-b}+\dfrac{x-b}{x-a}+2=0$

$(x-a)^2+2(x-a)(x-b)+(x-b)^2=0$

$\overline{x-a}+\overline{x-b}=0$

$x=\dfrac{a+b}{2}.$

(*b*) $x+y=a^2\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$ (1)

$\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{b}+\dfrac{b}{xy}$ (2)

$x+y=\dfrac{a^2}{xy}(x+y)$

I. ∴ $x+y=0$ $x=-y$

$1=\dfrac{a^2}{xy},\ y=\dfrac{a^2}{x}.$

Substituting this in 2nd

$x+\dfrac{a^2}{x}=\dfrac{a^2+b^2}{b}$

$x^2-\dfrac{a^2+b^2}{b}x=-a^2$

$\therefore x=\dfrac{a^2}{b}$ or b, $y=b$ or $\dfrac{a^2}{b}.$

8. Let $x=$ rate of steam launch, $y=$ distance from A to B, $\dfrac{y}{6}=$ time boat takes to go from B to A,

$\dfrac{y}{x+2\frac{1}{2}}+\dfrac{y}{x-2\frac{1}{2}}=$ time steam launch takes to go from and return to A.

$$\therefore \frac{y}{6}=\frac{y}{x+2\frac{1}{2}}+\frac{y}{x-2\frac{1}{2}}$$

$$x^2-\frac{25}{4}=12x$$

$x=12\frac{1}{2}$ miles per 1°.

9. Given $\alpha+\beta=a$ and $\alpha\beta=b^2$, find value of $\alpha^6+\alpha^4\beta^2+\alpha^2\beta^4+\beta^6$.

Expression $=(\alpha^4+\beta^4)(\alpha^2+\beta^2)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=a^2-2b^2$

$\alpha^4+\beta^4=(\alpha^2+\beta^2)-2\alpha^2\beta^2=(a^2-b^2)^2-2b^4=a^4-4a^2b^2+2b^4.$

∴ Expression $=(a^2-2b^2)(a^4-4a^2b^2+2b^4)$
$=a^6-6a^4b^2+10a^2b^4-4b^6.$

10. Show that the sum of the squares of the first n natural numbers is $n(n+1)(2n+1)\div 6.$

See Todhunter's Algebra.

Find an expression for the sum of the squares of the first n odd numbers,

$$\overline{2n-1}^3-\overline{2n-3}^3=6(2n-1)^2-12(2n-1)+8$$
$$\overline{2n-3}^3-\overline{2n-5}^3=6(2n-3)^2-12(2n-3)+8$$
$$\overline{2n-5}^3-\overline{2n-7}^3=6(2n-5)^2-12(2n-5)+8$$