the terms, $\phi_1, \phi_2, \ldots\ldots, \phi_{nr}$, are all the unequal cognate functions of $f(p)$, obtained by giving definite values to all the surds in $f(p)$ which are present in the coefficients of the powers of x in F, and forming the cognate functions without reference to the surd character of the surds thus rendered definite: F being understood to be generated directly by the multiplication together of the factors, $F_1(x)$, $F_2(x)$, &c., and to have the coefficients of the various powers of x arranged so as to satisfy the conditions of Def. 8.

For, all the terms in the series,

$$\phi_1, \phi_2, \ldots\ldots\ldots, \phi_n, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

are (by hypothesis) unequal. Suppose, if possible, that the terms,

$$\phi_{n+1}, \phi_{n+2}, \ldots\ldots\ldots, \phi_{2n}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

which are the roots of the equation, $F_2(x) = 0$, are not all unequal. Then, $F_2(x)$, having equal factors, has a measure, H, of less dimensions, as respects x, than $F_2(x)$, and yet involving, in the coefficients of the powers of x, merely such surds as occur in $F_2(x)$. But the surds in $F_2(x)$ are identical with those in $F_1(x)$. [For instance, let $F_1(x) = (1 + \sqrt{p})^{\frac{1}{3}}$, and, $F_2(x) = z(1 + \sqrt{p})^{\frac{1}{3}}$, where z is a third root of unity, distinct from unity. The presence of z in $F_2(x)$ does not affect the surds in the expression]. Therefore the expression H, of less dimensions as respects x than $F_1(x)$, involves in the coefficients of the powers of x merely such surds as appear in $F_1(x)$: which, [since $F_1(x)$ is the product of the terms, $x-\phi_1, \ldots\ldots, x-\phi_n$, where $\phi_1, \phi_2, \ldots\ldots \phi_n$, are all the unequal cognate functions of $f(p)$ obtained by assigning definite values to certain surds in $f(p)$], is (Cor. 4, Prop. VI.) impossible. Therefore all the terms in (2) are unequal. Next suppose, if possible, that some term in (2) is equal to a term in (1). Then $F_2(x)$ and $F_1(x)$ have a common measure; and their H. C. M. involves only such surds as appear in $F_1(x)$ or $F_2(x)$; that is, only such as appear in $F_1(x)$: which, as above, is (Cor. 4, Prop. VI.) impossible, unless $F_1(x)$ and $F_2(x)$ are identical. Suppose then, if possible, that $F_1(x) = F_2(x)$. The coefficients of like powers of x must be equal. Let the coefficient of a certain powers of x in $F_1(x)$, arranged according to the powers of y_1, (we choose a coefficient where y_1 occurs in some of its powers), and satisfying (as, by hypothesis, it does) the conditions of Def. 8, be,