SCHOOL WORK.

MATHEMATICS.

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Problems.—Honors.

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1. The lines drawn from the centre of the circle described about a triangle to the angular points are perpendicular to the sides of the triangle formed by joining the feet of the perpendiculars of the original triangle.

2. A straight line is drawn from the vertex A of the triangle ABC through the middle point of the base to a point D below the base; the lines AB, CD are produced to meet in P, and the lines AC, BD in Q. Prove that PQ is parallel to BC.

3. Prove that the factors of the sum of the squares of two numbers prime to each other, are themselves the sum of two squares.

4. If A, B, C be the angles of a triangle, prove that

$$\frac{\cot A+\cot B}{\cot\frac{1}{2}A+\cot\frac{1}{2}B}+\frac{\cot B+\cot C}{\cot\frac{1}{2}B+\cot\frac{1}{2}C}+\frac{\cot C+\cot A}{\cot\frac{1}{2}C+\cot\frac{1}{2}A}=1.$$

5. Sum the following series to infinity:—

(1) $$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{5\cdot 6\cdot 7}+\frac{1}{9\cdot 10\cdot 11}+\ldots=\tfrac{1}{4}\log 2.$$

(2) $$\frac{1}{1\cdot 3\cdot 5}+\frac{1}{5\cdot 7\cdot 9}+\frac{1}{9\cdot 11\cdot 13}+\ldots=\tfrac{1}{4}\left(\frac{\pi}{4}-\frac{1}{2}\right).$$

6. Prove that the continued fraction,

$$\frac{\sin\theta}{\cos\theta-}\,\frac{1}{2\cos\theta-}\,\frac{1}{2\cos\theta-}\,\&c.=\frac{\sin n\theta}{\cos n\theta}$$

if $\dfrac{1}{2\cos\theta}$ is repeated $(n-1)$ times.

7. Given $\tan^{-1}x^2+\tan^{-1}x=\tan^{-1}\frac{1}{8}$, find x.

8. A circle is inscribed in the triangle ABC touching the sides in the points D, E, F. If $l+m$, $m+n$, $n+l$, be the lengths of the sides of the triangle, show that $2\triangle ABC+\triangle DEF=\frac{1}{2}(lm+mn+nl)(\sin A+\sin B+\sin C)$.

9. Prove the following formulas:—

(1) When n is even

$$\frac{1}{\lfloor\underline{1}\,\lfloor\underline{2n-1}}+\frac{1}{\lfloor\underline{3}\,\lfloor\underline{2n-3}}+\ldots+\frac{1}{\lfloor\underline{n-1}\,\lfloor\underline{n+1}}=\frac{2^{2n-2}}{\lfloor\underline{2n}}.$$

(2) When n is odd

$$\frac{2}{\lfloor\underline{1}\,\lfloor\underline{2n-1}}+\frac{2}{\lfloor\underline{3}\,\lfloor\underline{2n-3}}+\ldots+\frac{1}{(\lfloor\underline{n})^2}=\frac{2^{2n-1}}{\lfloor\underline{2n}}.$$

10. Two debts are incurred, one of P with interest at rate r per annum, the other of P' with interest at rate r' per annum. The whole amount is to be paid off by equal instalments of M a month, covering principal and interest. Show that the number of payments to be made will be very nearly

$$=\frac{\log\left(1-\dfrac{P+P'}{M}\left\{(1+\rho)^{\frac{1}{12}}-1\right\}\right)}{-\frac{1}{12}\log(1+\rho)}$$

where $\rho=\dfrac{Pr+P'r'}{P+P'}$.

11. If α and β be the extreme, and γ the mean, angles of a harmonic pencil of four lines, prove that

(1) $\cos(\alpha+\beta)=\cos(\beta+\gamma)\cos(\gamma+\alpha)$.

(2) $2\tan\alpha\tan\beta\tan\gamma\tan(\alpha+\beta+\gamma)$
$=\tan(\alpha+\beta+\gamma)\tan\gamma-\tan\alpha\tan\beta$.

12. AB is the double ordinate to the axis of a given parabola, BR is a diameter, AR any line cutting the curve in Q; then if AP be taken in AR equal to QR, prove that the locus of P is a parabola.

13. If PQ is a chord of a central conic, normal at P, and if the normal at Q meets the tangent at P in N, and Y is the foot of the perpendicular on the tangent from its