and $\therefore\ \alpha+\beta+\gamma=$ a constant. q.e.d. As the point O can travel over the whole area ofthe triangle, it must also be allowed, under some conditions, to pass without the triangle, unless some impossibility is thus introduced. But whether O be within or without, the perpendiculars α, β, γ, remain real and no impossibility is introduced.

Therefore the statement that the algebraic sum of the perpendiculars from O to the sides of the triangle, is constant must hold for all positions of O in the plane of the triangle.

Now, if we enquire as to what takes place when the point O crosses a side of the triangle, the side a, or BC for example, we find the perpendicular α diminishes until it becomes zero, and then re-appears measured in an opposite direction. The algebraical way of expressing such a change is by a change of sign; and the perpendicular is said to change sign or to change sense, and this change transforms an addition into a subtraction, or *vice versa.* And with this convention our theorem is universally true.

In a similar manner. and by similar extensions, we have the statement $a\alpha+b\beta+c\gamma=$ constant, for every triangle.

This principle of moving or transforming a geometrical figure so as to vary the relative position of its parts while leaving its distinctive relations unaffected, except as to such ideas as change of sense, is an exceedingly important one, and introduces into geometrical methods a freedom of operation which was totally unknown in ancient geometry, and which, accordingly, has no illustration in Euclid's work.

I would advise the student to consider the following problems in this light.

1. In Euclid II, 4, by moving the point on the diagonal along the diagonal, until it passes beyond an end-point of the diagonal, prove from the original proposition that $(AB-BC)^2 = AB^2+BC^2-2AB.BC$; and that this is the 7th.

2. In Euclid II., 5, let AB be the line bisected in C, and let D be the point dividing AB unequally.

By moving D along until it passes A or B, and transforming the figure accordingly, show that the 5th contains the 6th, and *vice versa.*

These two propositions may be stated as one by modernizing the language employed. Thus, if A, C, B, be three equidistant point in line, and D be any fourth points in the same line, the rectangle on AD and DB is equal to the difference of the squares on CD or CB.

3. In Euclid II., 9, by moving the variable point D along the line AB until it passes beyond A or B, show by properly modifying the figure that props. 9 and 10 are one and the same, with the simple variation of internal or external division, or of taking a segment in different sense.