Another way of finding a, c, e, θ, ϕ *and* h.

§11. The values of a, c, e, θ, ϕ and h may be arrived at in another way. Let B and $B'\sqrt{z}$ be found as in §9. Then, because (see §9)

$$hz + h\sqrt{z} = (B + B'\sqrt{z})^2 - (a\sqrt{z})^5,$$

we have

$$hz = B^2 + (B'\sqrt{z})^2,$$

and

$$h\sqrt{z} = 2B(B'\sqrt{z}) - (a\sqrt{z})^5.$$

Here the quantities to which hz and $h\sqrt{z}$ are equated are known. Therefore h and z are known. When z is known, because a^2z and $\frac{c}{a}$ are known, a and c are known. Finally, θ and ϕ are obtained, as in §10, from the equations (26).

§12. *First Example.*—To exemplify the theory, let us take the equation

$$x^5 + 3x^2 + 2x - 1 = 0. \tag{27}$$

Because $k = -\frac{p_3}{20} = -\frac{3}{20}$, $p_4 = 2$, $p_5 = -1$, the equations (11) and (16) become

$$\left.\begin{aligned} &\frac{6t^2y}{5} + t\left(\frac{4y}{5} - 50y^2\right) = y + \frac{27}{1000}, \\ &t^2\left(\frac{125^2}{9}y^4 - \frac{500y^3}{9} + \frac{4y^2}{9} - \frac{9y}{100}\right) + t\left(\frac{625y^3}{9} + \frac{583}{180}y^2 - \frac{3y}{50}\right) \\ &= \frac{25y^3}{16} - \frac{38y^2}{45} - \frac{13y}{200} - \frac{81}{40000} \end{aligned}\right\} \tag{28}$$

The equation (19), obtained by eliminating t from the equations (28), is

$$\begin{aligned} F(y) = \left(\frac{25\times125^4}{36}\right)y^6 - \left(\frac{125^4}{9}\right)y^5 + \left(\frac{37\times125^3}{18}\right)y^4 - \left(\frac{1241\times125^2}{90}\right)y^3 \\ + \left(\frac{3209\times125}{36}\right)y^2 - \left(\frac{5159}{136}\right)y + \frac{109^2}{22500} = 0, \end{aligned}$$

and this has the commensurable root $\frac{1}{125}$. Therefore

$$y = a^2z = \frac{1}{125}.$$

Hence, from the two equations (28),

$$t = \frac{c}{a} = \frac{7}{4}.$$

In subsequent examples, when y and t have been found, we shall proceed at once to find B and $B'\sqrt{z}$, as in §9, without inquiring what a, c, e, θ, ϕ and h are separately. But we desire to illustrate, in one instance, the method of