and the angle AGB to DFE. But since AC is equal to DF, AG is equal to AC: and therefore the angle ACG is equal to the angle AGC, which is also an acute angle. But because AGC, AGB are together equal to two right angles, and that AGC is an acute angle, AGB must be an obtuse angle; which is absurd. Wherefore, BC is not unequal to EF, that is, BC is equal to EF, and also the remaining angles of one triangle to the remaining angles of the other.

Secondly. Let the angles ACB, DFE, be both *obtuse angles*. By proceeding in a similar way, it may be shewn that BC cannot be otherwise than equal to EF.

If ACB, DFE be *both right angles:* the case falls under Euc. I. 26.

Prop. XXVII. Alternate angles are defined to be the two angles which two straight lines make with another at its extremities, but upon opposite sides of it.

When a straight line intersects two other straight lines, two pairs of alternate angles are formed by the lines at their intersections, as in the figure, BEF, EFC are alternate angles as well as the angles AEF, EFD.

Prop. XXVIII. One angle is called "the exterior angle," and another "the interior and opposite angle," when they are formed on the same side of a straight line which falls upon or intersects two other straight lines. It is also obvious that on each side of the line, there will be two exterior and two interior and opposite angles. The exterior angle EGB has the angle GHD for its corresponding interior and opposite angle: also the exterior angle FHD has the angle HGB for its interior and opposite angle.

Prop. XXIX is the converse of Prop. XXVII and Prop. XXVIII.

As the definition of parallel straight lines simply describes them by a statement of the negative property, that they never meet; it is necessary that some positive property of parallel lines should be assumed as an axiom, on which reasonings on such lines may be founded.

Euclid has assumed the statement in the twelfth axiom, which has been objected to, as not being self-evident. A stronger objection appears to be, that the converse of it forms Euc. I. 17; for both the assumed axiom and its converse, should be so obvious as not to require formal demonstration.

Simson has attempted to overcome the objection, not by any improved definition and axiom respecting parallel lines; but, by considering Euclid's twelfth axiom to be a theorem, and for its proof, assuming two definitions and one axiom, and then demonstrating five subsidiary Propositions.

Instead of Euclid's twelfth axiom, the following has been proposed as a more simple property for the foundation of reasonings on parallel lines; namely, "If a straight line fall on two parallel straight lines, the alternate angles are equal to one another." In whatever this may exceed Euclid's definition in simplicity, it is liable to a similar objection, being the converse of Euc. I. 27.

Professor Playfair has adopted in his Elements of Geometry, that "Two straight lines which intersect one another cannot be both parallel to the same straight line." This apparently more simple axiom follows as a direct inference from Euc. I. 30.

But one of the least objectionable of all the definitions which have been proposed on this subject, appears to be that which simply expresses the conception of equidistance. It may be formally stated thus: "Parallel lines are such as lie in the same plane, and which neither recede from, nor approach to, each other." This includes the con-