

serve equal division of the circumference. But $\frac{1}{2}$ is excluded because it does not approximate $\frac{1}{3}$. Two-fifths is accepted as a third approximation. One-sixth is excluded, as $\frac{1}{3}$ is, for not approximating to numbers already found; $\frac{1}{2}$ and $\frac{2}{3}$ are excluded for being less than $\frac{1}{3}$, and therefore not sufficiently approximating $\frac{1}{3}$; $\frac{2}{5}$ approximates $\frac{1}{3}$, but is excluded because it is larger than $\frac{1}{3}$, and therefore does not come near enough to $\frac{1}{3}$; $\frac{3}{8}$ is accepted as a fourth approximation. The like reasoning would exclude all fractions not found in the series $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}$, etc., etc., which continually approximate $\frac{1}{3}$ ($\sqrt{3}$).

THIRD SOLUTION.—Using A, B and C in the sense of the first solution, it may be shown as there that $\frac{A}{2A+C}$ is between $\frac{1}{2}$ and $\frac{1}{3}$, put it therefore equal to $\frac{1}{2+\frac{1}{n}}$. But C must be more than half A

in order to approximate a division into thirds, therefore n is less than two, or of the form $1+\frac{1}{m}$. But when the third step is taken, overlapping the original starting point by D, we must have D approximate in value C, and therefore be more than $\frac{1}{2}$ C; therefore $m=1+\frac{1}{p}$. The like reasoning, by the aid of a simple diagram, would show that p is less than 2; and, thus continuing, we should show that the step A must equal a continued fraction whose first denominator is 2, and each subsequent denominator 1, which is easily shown to be equal to $\frac{1}{2}$ ($3-\sqrt{5}$).

This problem was proposed and solved by Mr. Chauncey Wright, in *Runkle's Mathematical Monthly*, and I have merely simplified and popularized the proof.

Botanists have known for nearly forty years that the leaves of plants are arranged by this law, and Mr. Wright's problem simply shows that the law gives the most rapid and thorough distribution of the leaves. Astronomers also show that the points of conjunction of the planets are scattered about the sun in the same manner; that is, by a law of time which is algebraically or intellectually the same with the law of spaces among leaves; and theology cannot fail to infer the reign of Intellect in the creation of plants and planets.

SELF-RELIANCE.

THE two great objects of intellectual education, are mental discipline and the acquisition of knowledge. The highest and most important of these objects is mental discipline, or the power of using the mind to the best advantage. The price of this discipline is effort. No man ever yet made intellectual progress without intellectual labour. It is this alone that can strengthen and invigorate the noble faculties with which we are endowed.

However much we may regret that we do not live a century later, because we can not have the benefit of the improvements that are to be made during the next hundred years, of one thing we may rest assured, that intellectual eminence will be attained during the 20th century just as it is in the 19th—by the *labour of the brain*. We are not to look for any new discovery or invention that shall supersede the necessity of mental toil; we are not to desire it. If we had but to supplicate some kind genius, and he would at once endow us with all the knowledge in the universe, the gift would prove a curse to us, and not a blessing. We must have the discipline of acquiring knowledge, and in the manner established by the Author of our being. Without this discipline our intellectual stores would be worse than useless.

The general law of intellectual growth is manifestly this;—whatever may be the mental power which we at any time possess, it requires a repetition of mental efforts, equal in degree to those which we have put forth before, to prevent actual deterioration. Every considerable step of advance from this point must be by a new and still higher intellectual performance.

There are many impediments in the path of the student, which it is desirable to remove; but he who attempts to remove all difficulties, or as many of them as possible, wars against the highest law of intellectual development. There can not be a more fatal mistake in education, than that of a teacher who adopts the sentiment, that his duty requires him to render the daily tasks of his pupils as easy as possible.

There is, perhaps no error in our schools at the present time more deeply seated or more widely extended than the ruinous practice of aiding pupils in doing work which it is all-important they should do for themselves. Our progress in the art of cultivating habits of earnest, independent thought, has not kept pace with our improvements in other departments of education. Familiar explanations, and illustrations, and simplifications, and dilutions, too often spare the pupil the labour of thinking for himself, and thus dwarf the intellect, and defeat the highest object for which our schools are established.

To secure from a pupil the solution of a difficult problem will often cost time which the teacher can ill afford; it may often cost

more effort to secure a solution from the pupil, than it costs the pupil to do the work. The pupil has tried the problem, and satisfied himself that he is not able to solve it; the teacher may be satisfied that the pupil can perform it, but if he can not make the pupil think so too, it will be difficult to bring his best energies to bear upon it; and even after the pupil is persuaded that he is able to accomplish the task, it may still be necessary for the teacher to adopt special measures to set the pupil's mind at work. The pupil may have the ability to solve the problem; he may believe that he has this ability; and he may have a willing mind; and, after all, fail entirely of doing it. And this brings to view what must be regarded as the highest gift of the teacher: namely, the ability to teach his pupils how to think and act, without doing their thinking and acting for them.

When a pupil has failed to overcome an obstacle, his mind may often be quickened to assist by requesting him to explain the steps he has taken. "Great thoughts," says Dr. Channing, "are never fully possessed till he who has conceived them has given them fit utterance." So with a pupil attempting to surmount a difficulty; the very effort required to express a thought in language often aids materially in grasping the thought itself.

A scholar had become discouraged over a difficult question. He had gone through the solution again and again, but could not obtain the answer sought. The teacher availed himself of a favourable opportunity, and requested the pupil to go through the work slowly and carefully in his presence. As the pupil proceeded the teacher required him to explain each step of the process; and when he reached the point where his previous error occurred, as the teacher asked him to give his reason, the pupil's eye flashed with delight and he exclaimed, "I see my mistake!" Without further assistance he soon reached a correct result. The teacher had not furnished the slightest hint in respect to the solution of the problem. He had only taken measures which brought the pupil's own strength to bear upon it.

There are, however, peculiar cases which no such method will reach. The pupil may be required to repeat his solution a hundred times, in the presence of the teacher or alone, with reasons or without, and all to no purpose. The result, if he reaches one, is sure to be wrong. It is not wise, even now, for the teacher to give over in despair. Let him ask the pupil such questions as will call to mind the principles which he has occasion to apply, and, in a majority of cases, the pupil will need no further aid.

The same end may usually be gained by giving the pupil an example involving the difficulty over which he has stumbled, but less complicated in other respects; or by giving him several examples, leading gradually to the main obstacle to be overcome. I believe the cases are exceedingly rare in which minds properly disciplined would ever be benefited by direct assistance, in an ordinary course of mathematical study. But if it be thought best, in extreme cases, to afford this assistance, let the pupil, by all means, be required to repeat the process, after the teacher's work has been entirely erased; and thus derive, at least, the benefit of reproducing, though he has not the power to originate.

The teacher will find it a highly useful exercise to give his pupils an occasional model of thinking. Let him take a problem to the blackboard, and think aloud as he proceeds with the solution; so that the pupils may witness the action of the teacher's mind, and observe the questions he asks himself, and the various associations and comparisons that arise, as he advances from step to step in the process.

I am aware that in many schools the teachers can not dwell upon particular points with the degree of thoroughness that I have recommended; but this does not affect the importance of the principle, which should be applied whenever the circumstances permit.

In most of our schools pupils indulge, to a greater or less extent, in the practice of assisting one another in the solution of difficult questions. I need not say that we should labour most assiduously to eradicate this injurious practice. Pupils should be taught to regard it as dishonourable, either to assist others or to receive assistance, except under the special cognizance and direction of the teacher.

Permit me, in this connection, to allude to one of the helps kindly furnished by a large class of publishers and authors, for the special benefit of teachers; but which many pupils have thought to be quite as well suited to their wants as to the wants of instructors. I refer to printed keys, containing solutions of all the more difficult problems in arithmetic and other branches of mathematics.

There are undoubtedly cases in which the time of the teacher is so limited that it is necessary for him to resort to the use of a key; but with pupils their effect is always injurious, sapping the very foundation of everything adapted to promote manly, independent thought. Even with teachers who are compelled to resort to the use of keys for the purpose of saving time, it must be confessed that the tendency of the practice is to render instruction superficial.

The practice of introducing young children to the study of English grammar as a science, and assigning them daily lessons to be prepared from a text-book, is exceedingly injurious in its influence upon their mental habits. A thorough and intelligent analysis of the structure of language is beyond the capacity of children eight or nine years of age.

Instruction in the use of language should be commenced as soon as children enter school, and all the primary classes should have