Let C = Weight of cement in grams.

p = Percentage of water for normal consistency of the cement.

A = Total surface area of the aggregate in sq. ins.

$N = A/C$ = square inches of surface area of aggregate per gram of cement.—Arbitrarily selected before the test.

w = Cubic centimeters of mixing water.

c = Volume of the cement in cc. = 0.67C.

As stated in (1),

$$w = pC + (A/210) \qquad (2)$$

From above,

$$A = NC \qquad (3)$$

Substituting (3) in (2),

$w/C = p + (N/210)$, in which p and N are constants.

To express the weight of the cement in volume measure, $w/0.67C = pN/210$ = constant, which is the water-cement ratio proposed by Professor Abrams for equal strength, or $w/c = K$.

Arrive at Same Conclusion

Expressing this result in words, the surface area theory, after fixing a ratio of cement to surface area of the aggregate, arrives at the same final conclusion as does the method proposed by Professor Abrams,—that strength depends only upon the ratio of mixing water to cement.

The fineness modulus theory states that strength is dependent only upon the water-cement ratio, and the surface area theory arrives at the same final conclusion after providing a relation between the quantity of cement and the surface area of the aggregate.

On any concrete work, it is necessary that the concrete have some minimum plasticity, consistency or flowability, in order that it may be placeable with a reasonable amount of work. For the same flowability, an aggregate containing much fine material will require a greater amount of mixing water than one which is coarse. Since it is generally agreed that any additional quantity of mixing water added to the same mix will result in lower strength, it is of the utmost importance that the laboratory accurately take into account this difference in water requirements, otherwise the relative water contents must be modified on the job to secure the necessary minimum workability, which in turn will result in concretes differing in strengths from those produced in the laboratory.

Lack of accurate means of measuring consistency of concrete mixtures has been a serious obstacle to the proper testing and study of the properties of concretes in the laboratory. It is unfortunate that there has been no means for properly determining and controlling the water requirements of concrete, since variation in water content has so great an effect on strength. The writer believes that had Prof. Abrams and Mr. Edwards been able to properly control and measure this requirement, neither of the proposed theories for the proportioning of concrete mixtures would have been presented to the public in their present form.

Must Have Same Consistencies

The proponents of both theories are agreed that concretes must have the same consistencies to be comparable. In Lewis Institute Bulletin No. 1, Prof. Abrams states that the 27 concretes shown in Table 2 had the same consistencies as measured by the cylinder slump test, the column of results marked 100% consistency having slumps of ½-in. to 1-in. as measured in this test.

Mr. Edwards in A.S.T.M. proceedings for 1918, page 253, states: "The marked influence of consistency of the mix upon the ultimate strength of mortars renders it especially important that test mortars be made of uniform consistency. The importance of this investigation as a preliminary to the making of tests tending to prove or disprove the validity of the primary theory of the surface area method of proportioning is self-evident." Following this statement Mr. Edwards gives the surface area water formula as shown above.

There is no question that both investigators fully recognize the basic requirement that equal consistencies must be obtained, but tests of their own aggregate gradings show that both failed to secure such comparable concretes in practice.

Eight of the aggregates included in Table 2, Lewis Institute Bulletin No. 1, were prepared by screening Potomac river sand and gravel and recombining them to conform with the sieving analyses shown in this table. Concrete was made of the proportion 1: 5, using equal quantities of mixing water in all as specified by the water formula described on page 13 of Bulletin No. 1.

Both the cylinder slump test and the vibrating steel plate, another method used in the laboratory for measuring flowability, indicated wide differences in consistency for these concretes. In two cases the cylinder form was withdrawn and the masses of concrete picked up by hand and carried back to the mixer without losing their shapes. Two other aggregates gave slumps of 7¾-ins. and 8½-ins.

Later tests showed that these concretes which were described as having equal consistencies, actually varied more than 25% in this respect; that is, the amount of water in the wettest of the mixes was more than 25% greater than that required to yield the consistency of the driest of the mixes. The differences in strength found for these mixes is of secondary importance, since concretes differing so widely in consistency are not comparable.

Wide Variations in Consistency

Equally wide variations in consistency were found among mortars made with aggregates having the same gradings as those used by Mr. Edwards. The three aggregates C, E, and G, in Table 7, page 256, of the above-mentioned A.S.T.M. proceedings, were reproduced. The quantity of water specified by the Edwards water formula was found to be entirely insufficient. The quantity was increased by a constant in order to obtain mixtures which had some degree of workability, and the resulting wide differences in consistency were very apparent to the eye and verified by the flowability test. The test data are shown below:—

MIXING WATER

Aggregate.	Group 1: Specified by the Edwards formula.		Group 2: As increased to obtain workable mix.			Group 3: As used to obtain equal flows.			Comp. strength, lbs. per sq. in.
	cc.	w/c.	cc.	w/c.	Rel. flow.	cc.	w/c.	Flow.	
E	553	.426	720	.555	170	720	.555	170	3,855
C	343	.426	447	.555	125	497	.616	168	3,170
G	178	.426	234	.555	No*	342	.820	171	1,810

*No flow; not plastic.

In Group 1 the quantity of mixing water specified by the "surface area" water formula was used. The quantity of water supplied was entirely inadequate to result in plastic workable mixes for the aggregates C and G. In Group 2 the quantity of water was increased proportionately for all aggregates, but G was still too dry to be workable, and the wide variation in consistency obtained for E and C was very marked. It was manifestly improper to mold mortars differing so greatly in consistency, so the flowabilities were made equal as shown in Group 3. This was done by increasing the quantities of mixing water for aggregates C and G.

This series of tests, which confirms others outlined to test the surface area theory, shows conclusively, that (1) the Edwards water formula, which results in a constant water-cement ratio (w/c), as proposed by Prof. Abrams, is entirely inadequate, and (2) the resulting mortars have wide differences in consistency, so that the resulting strength values are not comparable. The addition of sufficient water to equalize consistencies furnishes mortars which are comparable, but fully discredits the proposed water formula, since the (w/c) relation is no longer constant. In this particular case, (w/c) varied from .555 to .820.

Using Standard Ottawa Sand

A comparison of the amount of water which the Edwards formula would furnish to a 1: 3 standard Ottawa sand mortar, as used in the routine testing of cement, is also interesting.

Assume the percentage of mixing water required for normal consistency of a given cement to be 24.