direction of an increase of l, contrary to the first instance. In this case we get the equation

$$\frac{L}{g}\frac{dv}{dt} + z - h = o \qquad (14)$$

For any case we may combine both equations into a fundamental formula:

$$\frac{L}{g} \cdot \frac{dv}{dt} + z \pm h = o \qquad (15)$$

in which the plus refers to the velocity from intake to surge tank and the minus sign for the reverse movement. If we now introduce for h a function, the value of which has the same sign as v, then the double sign may be dropped and the whole movement may be represented by the equation:

$$\frac{L}{g} \cdot \frac{dv}{dt} + z + h = o \qquad (16)$$

A second fundamental equation may be developed from the condition of continuous flow, that is to say, the volume of water which flows to the surge tank in the time element dt must be equal to the sum of the changes of volume in the surge tank and in the volume flowing out of the tank at the same time ∴

$$v.a.dt = s\,A\,dt + q.dt.$$
$$a.v = As + q = A(s + c) \qquad (17)$$

The questions which are of principal interest are:

1st. How is the movement of the water surface in the surge tank with regard to time affected by given dimensions of the main conduit and the surge tank and given conditions as to overflow?

2nd. What dimensions must the surge tank have to agree with given conditions as to the main conduit and outflow, in order that the fluctuation of water level does not exceed certain amounts, determined by local conditions? The following cases will be investigated:

(a) Sudden partial or entire cessation of the outflow.

(b) Sudden starting of the outflow.

(c) Gradual cessation, or starting, variable outflowing conditions; and

(d) Influence of a spillway built in the surge tank.

The formulæ will be developed in the first case for a constant sectional area of surge tank, and under the assumption that h is proportional to v, i.e., $h = n.v$.

It will be shown that it is possible to investigate all cases according to a uniform method, analytically and graphically, with the assistance of the well-known theory of damped and forced undulations. The inaccuracies which result because h is proportional to v^2 and the corrections which must be applied to the results of the first method will be shown in the final study. With the simplifying assumptions already mentioned, we get from equation (17)

$$v = \frac{A}{a}(s + c) \qquad \frac{dv}{dt} = \frac{A}{a}\left(\frac{ds}{dt} + \frac{dc}{dt}\right) \qquad (18)$$

This applied in equation (16) and the whole equation divided by

$$T^2 = \frac{L}{g} \cdot \frac{A}{a} \ (19), \text{ we get } \frac{ds}{dt} + \frac{dc}{dt} + \frac{z}{T^2} + \frac{h}{T^2} =$$

$$\frac{ds}{dt} + \frac{n.A}{T^2.\,a} \cdot s + \frac{z}{T^2} + \frac{n\,A}{T^2 a} \cdot c + \frac{dc}{dt} = o \qquad (20)$$

If we introduce as an abbreviation:

$$T_o = \frac{T^2.\,a}{n\,.\,A} \qquad (21)$$

and consider that

$$s = \frac{dz}{dt} \qquad \frac{ds}{dt} = \frac{d^2z}{dt^2} \qquad (22)$$

then follows the principal equation:

$$\frac{d^2z}{dt^2} + \frac{1}{T_o} \cdot \frac{dz}{dt} + \frac{z}{T^2} + \frac{c}{T_o} + \frac{dc}{dt} = o \qquad (23)$$

$$\text{The values } n \text{ and } T = \sqrt{\frac{A\,L}{a.g}}$$

$$\text{and } T_o = \frac{T^2}{n} \cdot \frac{a}{A} = \frac{L}{n.g} \qquad (24)$$

are times with regard to their dimensions. Starting with this principal equation, we may investigate the different cases as follows:

III. Special Cases.

Case A.—Sudden Shut-down.

Preceding a shut-down, Q_1 cubic feet per second flows out of the surge tank. During the normal condition in the main conduit, the velocity $= v_1 = \frac{Q_1}{a}$ wherefore $q = Q_1$. The water surface in the surge tank is h_1 feet lower than the static level n—n. The time t is measured from the beginning of the shut-down, therefore from $t = o$, q becomes ϵq if ϵ is the proportion of the steady flow subsequent to the shut-down in relation to the flow preceding the shut-down.

After the sudden shut-down, the following phenomena occur in the surge tank: The water surface rises with variable velocity until it reaches a maximum height. When the highest elevation is reached, the reverse movement occurs. The velocity increases as the water level recedes, then decreases until the lowest level is reached, after which an ascending movement occurs but to a somewhat less height than before, and so on, until the normal conditions with the constant flow $\epsilon . Q_1$ become established. The movement of the water level belongs in the category of the damped oscillations.

(1) Analytical Investigation.—The formula (23) for this case, under the condition that

$$c = \frac{q}{A} = \frac{\epsilon\, Q_1}{A} = \epsilon\, c_1 = \text{a constant } \left(\frac{dc}{dt} = o\right) \qquad (25)$$

becomes

$$\frac{d^2z}{dt^2} + \frac{1}{T_o}\frac{dz}{dt} + \frac{z}{T^2} + \epsilon\frac{c_1}{T_o} = o \qquad (26)$$

Since $h_1 = n.v_1 = n\,c_1 \dfrac{A}{a} = c_1 \dfrac{T^2}{T_o}$, it follows that