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BENDING MOMENTS IN FLAT SLABS

A MODIFICATION OF GRASHOF'S GENERAL BENDING MOMENT THEORY, FOR CONSIDERATION OF UNIFORMLY DISTRIBUTED LOAD—THE LACK OF SUFFICIENT REINFORCEMENT A COMMON DEFECT IN MANY FLAT SLAB SYSTEMS

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WITH very few exceptions, the flat slab systems in use show lack of adequate reinforcement. The designs are very similar, as regards insufficient reinforcing, to those adopted during the period 1885 to 1895 in the making of ordinary slabs and beams. As long as the tension was at the bottom side of a structure the steel was placed correctly, and in general it was also correctly placed over columns, but for slightly more complicated structures and stress distributions but little understanding was shown. A great number of "systems" existed, formed, as a rule, by adding a few bars, here and there, to the usual reinforcing. As these "systems" were able to carry a test load of as much as twice the live load without failing and with a deflection of only a small fraction of an inch, their inventors considered these structures to be a complete success and assured architects and others interested in buildings of strength, rigidity, economy, etc., of the inventions, by means of neat printed matter and pictorially illustrated advertisements, showing a pile of well filled bags resting on the structure. There was usually added a list giving the various jobs where the "system" had been employed. The excellence of the invention was thereby made clearly evident without question.

To-day theoretical and practical experience will veto most of these early "systems." The same veto should be placed on the majority of the flat designs now in use.

The thorough and scientific test loadings, made by the University of Illinois (Prof. Talbot) and others, of flat slab floors in actual structures have shown the insufficiency of their reinforcing in a practical way. Below it is shown theoretically.

In nearly all flat slab designs it is found that the positive bending moments and the negative moment over the columns are provided for, more or less successfully. The existence of these moments are, of course, easily recognizable, even by those whose knowledge of mechanics does not extend much beyond the stresses and strains in a simply supported girder or truss. But the negative moments perpendicular to the sides of the panels are, as a rule, entirely neglected, although they have about the same numerical value as the maximum positive moments.

The following calculations are based on Grashof's general theory* modified by the general assumptions on which all calculations for reinforced concrete depend. The method adopted follows one given in a recent issue of "Deutsche Bauzeitung."

For an infinite slab, of uniform thickness, h, supported at a series of points, dividing it into square panels of side, 2a, and uniformly loaded with q per sq. unit, Grashof gives the following values for the deflections, z, at the various points x, y:

$$z = \frac{g}{4 \,.\, A \,.\, h^3} \,.\, [(a^2 - x^2)^2 + (a^2 - y^2)^2]$$

where $A = \frac{m^2}{m^2 - 1} \,.\, E$, m is Poisson's constant,

and E the modulus of elasticity.

The origin of the system of co-ordinates lies at the middle point of the diagonal of the panel, the directions of the x and y axes being parallel to the sides.

The horizontal normal stresses c_x and c_y at the point x, y, v (that is, the point whose distance is v from the centerplane of the slab) are

$$c_x = -A \,.\, v \,.\, \left(\frac{\delta^2 z}{\delta x^2} + \frac{1}{m} \,.\, \frac{\delta^2 z}{\delta y^2}\right)$$

and

$$c_y = -A \,.\, v \,.\, \left(\frac{1}{m} \,.\, \frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2}\right)$$

From the equation for z it follows that

$$\frac{\delta^2 z}{\delta x^2} = \frac{g}{A \,.\, h^3} \,.\, (3x^2 - a^2)$$

and

$$\frac{\delta^2 z}{\delta y^2} = \frac{g}{A \,.\, h^3} \,.\, (3y^2 - a^2)$$

$$\therefore \quad c_x = -\frac{g}{h^3} \,.\, v \,.\, \left[3x^2 - a^2 + \frac{1}{m} \,.\, (3y^2 - a^2)\right]$$

and

$$c_y{}^2 - \frac{g}{h^3} \,.\, v \,.\, \left[\frac{1}{m} \,.\, (3x^2 - a^2) + 3y^2 - a^2\right].$$

The corresponding bending moments per unit width are

$$M_x = -\frac{g}{h^3} \,.\, \frac{h}{2} \,.\, \left[3x^2 - a^2 + \frac{1}{m} \,.\, (3y^2 - a^2)\right] \,.\, \frac{h^2}{6}$$

$$M_y = -\frac{g}{h^2} \,.\, \frac{h}{2} \,.\, \left[\frac{1}{m} (3x^2 - a^2) + (3y^2 - a^2)\right] \,.\, \frac{h^2}{6}$$

or

*Theorie der Elastizität und Festigkeit, Berlin, 1878, page 358.