THE FLOW OF WATER IN SIPHONS*

By Mark Halliday

An analysis of the flow of water in siphons, simple and compound, suggested itself to the writer after the perusal of the discussion of Mr. George R. Nicholson's paper on "The Horsley and Nicholson Automatic Compound Siphon."

Consider first the simple siphon shown diagrammatically in Fig. 1, which is arranged to siphon water from the tank A over the point B to the tank C.

Let H_1 = static head in tank A above datum;
H_2 = static head at point B above datum;
H_3 = static head in tank C above datum;
p_1 = pressure in pipe at entrance A;
p_2 = pressure in pipe at point B;
p_3 = pressure in pipe at exit C;
v_1 = velocity of water at entrance to pipe at A;
v_2 = velocity of water in pipe at point B;
v_3 = velocity of water in pipe at exit C.

Then, by Bernouilli's theorem—

$$\left.\begin{aligned} & H_1 + \frac{p_1}{62{\cdot}4} + \frac{v_1^2}{2g} \\ = {} & H_2 + \frac{p_2}{62{\cdot}4} + \frac{v_2^2}{2g} + \text{friction head from A to B} \\ = {} & H_3 + \frac{p_3}{62{\cdot}4} + \frac{v_3^2}{2g} + \text{friction head from A to C} \end{aligned}\right\} \quad (1)$$

The friction head in feet $\frac{4flv^2}{2gd}$ for round pipes - (2)

where f = coefficient of friction; l = length of pipe, in feet; v = velocity of water, in feet per second, and d = diameter of pipe, in feet.

Transposing equation (1) gives—

$$H_2 - H_1 = \frac{p_1}{62{\cdot}4} + \frac{v_1^2}{2g} - \frac{p_2}{62{\cdot}4} - \frac{v_2^2}{2g} - \frac{4fl_1v_2^2}{2gd} \quad (3)$$

l_1 being the length of pipe from A to B, the loss at entry to the pipe is neglected.

Also, if the pipes are of equal diameter throughout, $v_1 = v_2$.

In order to obtain the maximum velocity at point B, $p_2 = 0$.

$$\text{Then} \quad H_2 - H_1 = \frac{p_1}{62{\cdot}4} - \frac{4fl_1v_2^2}{2gd} \quad \text{- - - -} \quad (4)$$

but $\frac{p_1}{62{\cdot}4}$ = head of water, in feet, equivalent to atmospheric pressure or, say, 34 ft.

$$\text{Then} \quad H_2 - H_1 = 34 - \frac{4fl_1v_2^2}{2gd} \quad \text{- - -} \quad (5)$$

$$\text{Let} \quad H_2 - H_1 = h_2 \quad \text{- - - - - - -} \quad (6)$$

$$\text{Then} \quad v_2 = \sqrt{\frac{2gd(34 - h_2)}{4fl_1}} \quad \text{- - - -} \quad (7)$$

$$\text{Also let} \quad H_1 - H_3 = h_3 \quad \text{- - - - -} \quad (8)$$

$$\frac{p_1}{62{\cdot}4} = \frac{p_3}{62{\cdot}4} = 34 \text{ ft.} \quad \text{- - - - - - -} \quad (9)$$

And $v_1 = v_3$ if the pipes are of equal diameter.

From equation (1)—

$$H_1 = H_3 + \frac{4fl_2v_3^2}{2gd} \quad \text{- - - - - -} \quad (10)$$

l_2 being the length of pipe from A to C.

$$\therefore v_3 = \sqrt{\frac{2gah_3}{4fl_2}} \quad \text{- - - - - - - - - -} \quad (11)$$

$$v_3 = v_2 \quad \text{- - - - - - - - - - - - - -} \quad (12)$$

$$\text{Then} \quad \frac{2gdh_3}{4fl_2} = \frac{2gd(34 - h_2)}{4fl_1} \quad \text{- - - - - -} \quad (13)$$

$$\text{or} \quad \frac{h_3}{l_2} = \frac{34 - h_2}{l_1} \quad \text{- - - - - - - - - - - -} \quad (14)$$

Unless this relationship holds, the siphon will not work continuously without a regulating valve. It will be noted that h_2 equally as much as h_3 governs the discharge. If h_3 is excessive, then v_3 tends to become larger than v_2, and cavitation in the pipes will result.

This explains the statement made by so many that some siphons work better when the valve at the delivery end is partly closed. This must necessarily be the case, as the valve must be regulated until (v_3 × area at C) = (h_2 × area at B).

The same reasoning when applied to the Nicholson compound siphon results in the following deductions:

Fig. 2 shows the diagrammatic arrangement of the siphon; H_2, $\frac{p_2}{62{\cdot}4}$, and v_2 are the static pressure, velocity and energies per pound of water respectively at the air inlet N of the compound siphon. Then if the air inlet and trap N, S, N, is fixed in a position according to the relationship in equation (14), *viz.*, such that $\frac{h_3}{l_2} = \frac{34 - h^2}{l_1}$ (14), the compound siphon will discharge as much water as any simple siphon.

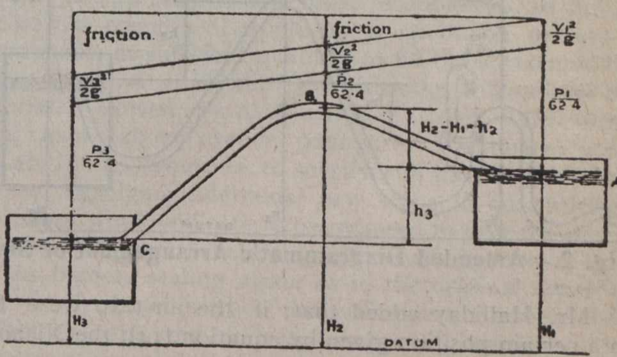

Fig. 1.—Single Siphon Shown Diagrammatically

It has been assumed throughout that f, the coefficient of friction, is the same for the whole length of pipe considered; also, in order to simplify the argument, that $p_2 = 0$.

For a maximum discharge, this would be so, but the analysis would hold equally well if p_2 had a value of a few feet.

In that case the figure could be inserted to slightly modify the result in equation (14).

Discussion.

Mr. G. R. Nicholson, whose paper had inspired Mr. Halliday's analysis, remarked that he wished Mr. Halliday had enlarged on the question of cavitation. Whenever a simple siphon was worked on a long length of pipe line, considerable friction occurred, and when an extensive length of pipe dropped a considerable depth below the level of the water at the intake, by the law of gravitation the velocity at the outlet would be greater than the velocity at the intake and cavitation occurred at the highest point of the siphon, i.e., a partial vacuum

*From a paper read before the North of England Institute of Mining Engineers.