each other in A and B, and from A a chord be drawn cutting them in C and D, prove that the part CD between the circumferences will be bisected by the circle described on AB as diameter.

Circles ABC, ABD are equal, and AB the common chord; CAD any line through A terminated by the circumference. Let E be the point where circle on AB cuts CD. Join BE, BD, BC,

$$\therefore \angle BEA = \frac{\pi}{2} = \angle BED. \quad 31, \text{III}.$$

Since the circles are equal and AB a common chord the $\angle ACB = \angle BDA$,

$\therefore$ in the two triangles BEC, BED the angles BEC, $ECB =$ angles BED, EDB, and BE common side,

$$\therefore CE = ED. \qquad 26, \text{I}.$$

4. Prove that

$$\frac{a^2\left(\frac{1}{b}-\frac{1}{c}\right)+b^2\left(\frac{1}{c}-\frac{1}{a}\right)+c^2\left(\frac{1}{a}-\frac{1}{b}\right)}{a\left(\frac{1}{b}-\frac{1}{c}\right)+b\left(\frac{1}{c}-\frac{1}{a}\right)+c\left(\frac{1}{a}-\frac{1}{b}\right)} = a+b+c.$$

Multiplying numerator and denominator by abc and arranging with regard to a

$$\frac{a^3(c-b)-a(c^3-b^3)+bc(c^2-b^2)}{a^2(c-b)-a(c^2-b^2)+bc(c-b)}$$

$$=\frac{a^3-a(c^2+bc+b^2)+bc(c+b)}{a^2-a(c-b)+bc} = a+b+c.$$

5. If $x+y+z=xyz$ prove that

$$\left(\frac{x}{y}+\frac{y}{x}+\frac{y}{z}+\frac{z}{y}+\frac{z}{x}+\frac{x}{z}+2\right)^2 = (1+x^2)(1+y^2)(1+z^2).$$

$$\text{1st side} = \left(\frac{x+z}{y}+\frac{y+z}{x}+\frac{y+x}{z}+2\right)^2 =$$

substituting for x

$$\left\{\frac{\frac{y+z}{yz-1}+z}{y}+\frac{y+z}{\frac{y+z}{yz-1}}+\frac{y+\frac{y+z}{yz-1}}{z}+2\right\}^2$$

$$=\left\{\frac{1+z^2}{yz-1}+yz-1+\frac{y^2+1}{yz-1}+2\right\}^2$$

$$=\left\{\frac{(1+z^2)(1+y^2)}{yz-1}\right\}^2 = \frac{(1+z^2)(1+y^2)}{(yz-1)^2}(1+z^2)(1+y^2)$$

but $\dfrac{(1+z^2)(1+y^2)}{(yz-1)^2} = \dfrac{1+z^2+y^2+y^2z^2}{y^2z^2-2yz+1}$

$$=1+\frac{1+z^2+y^2+y^2z^2-y^2z^2+2yz-1}{(yz-1)^2}$$

$$=1+\left(\frac{z+y}{yz-1}\right)^2 = 1+x^2,$$

$$\therefore \text{1st side} = (1+x^2)(1+y^2)(1+z^2).$$

The following is another solution of same question by A. MacMurchy, University College:

$$\text{Sinister} = \left(\frac{xyz-x}{x}+\ldots+\ldots+2\right)^2$$

$$=1+y^2z^2+z^2x^2+x^2y^2+2xyz(x+y+z)-2yz-2zx-2xy.$$

$$=1+y^2z^2+z^2x^2+x^2y^2+x^2y^2z^2+(x+y+z)^2-2yz-2zx-2xy$$

$$=1+x^2+y^2+z^2+y^2z^2+z^2x^2+x^2y^2+x^2y^2z^2$$

$$=(1+x^2)(1+y^2)(1+z^2).$$

7. A waterman rows a given distance a and back again in b hours, and finds that he can row c miles with the stream in the same time as d miles against it. Find the time each way, and the rate of the stream.

Let $x=$ rate of rowing.
$y=$ " stream.
$z=$ time it takes to row down stream.

(1) $(x+y)z=a$.

(2) $(x-y)(b-z)=a$.

(3) $\dfrac{c}{x+y}=\dfrac{d}{x-y}$.

from (3) $\dfrac{x}{y}=\dfrac{c+d}{c-d}$

$$\frac{x+y}{y}=\frac{2c}{c-d} \quad \frac{x-y}{y}=\frac{2d}{c-d}$$

dividing (1) $\dfrac{z}{y}=\dfrac{a(c-d)}{2c}$

dividing (2) $\dfrac{b-z}{y}=\dfrac{a(c-d)}{2d}$,

$$\therefore \frac{z}{b-z}=\frac{d}{c} \quad z=\frac{bd}{c+d}$$

$$b-z=\frac{bc}{c+d} \quad y=\frac{2bcd}{a(c^2-d^2)}.$$

8. ABC is an isosceles triangle; D the middle point of the base BC. If any straight line drawn through D meets one side in E