

13. Page 196, line 7. Two lumps of ice of the same size, one of them wrapped in flannel, are lying in a pan in a warm room. Which one will melt more quickly? Why? (6)

Lesson LXXXI., page 227.

14. What question does the poet ask the waterfowl in two different stanzas on this page 227? Point out the differences in the way in which it is asked, comparing one stanza with the other. (3, 4)

15. Line 5. *Do the wrong*; in what way? (3)

16. Line 7. What preceding phrase is the equivalent of *On the crimson sky*? (3)

17. Line 9. What does the word *plashy* suggest? (3)

18. Line 10. Why *marge* instead of *margin*? (3)

19. Line 11. How does *rocking* describe the billows? (3)

20. Page 228, line 15. "Hath sunk the lesson." What lesson does the poet mean? Tell it fully in your own words. (6)

Lesson XCIII., page 272.

24. Use words as different as you can from those in the book and avoid the use of the 1st personal pronoun to tell the statements contained in the 2nd, 3rd and 4th stanzas.

Maximum 103 marks; 100 marks a full paper; 33 minimum to pass.

Full value ought not to be given for any answer unless it is carefully written in a correct, complete sentence, correctly spelled.

ALGEBRA.

Solutions by S. A. MITCHELL, Queen's Col.

1. Given the expression $\frac{\Delta}{2} \left\{ 2 + \frac{a}{s-a} + \frac{b}{s-b} + \frac{c}{s-c} \right\}$ as the area of the triangle whose vertices are the excentres of the triangle with sides, a, b, c , where $2s = a + b + c$, and $\Delta^2 = s(s-a)(s-b)(s-c)$; if $4\Delta R = abc$, show by algebraical reduction that this area is $2 R_s$.

$$\begin{aligned} 1. \quad & \frac{\Delta}{2} \left\{ 2 + \frac{a}{s-a} + \frac{b}{s-b} + \frac{c}{s-c} \right\} \\ &= \frac{\Delta}{2} \left\{ 2s^4 - (a+b+c)s^3 + abc s \right\} \\ &= \frac{\Delta}{2} \left\{ \frac{2s^4 - 2s^4 + 4\Delta R_s}{s(s-a)(s-b)(s-c)} \right\} = \frac{\Delta}{2} \frac{4\Delta R_s}{\Delta^2} \\ &= 2 R_s. \end{aligned}$$

2. (a) Explain and establish any two principles upon which we may proceed in endeavoring to factor an expression.

(b) The sum of the fourth powers of a number, of its reciprocal, and of 1, may be expressed as the product of four factors, each involving the number and its reciprocal.

2. (a) (i). If we can put the expression into the form of the difference between two squares, we can factor it directly, since $a^2 - b^2 = (a-b)(a+b)$ whatever a and b may stand for.

$$\begin{aligned} \text{Thus: } 3x^2 - 2x + 4 &= \frac{1}{3} (9x^2 - 6x + 12) \\ &= \frac{1}{3} (9x^2 - 6x + 1 - 11) = \frac{1}{3} \left\{ (3x-1)^2 - \sqrt{-11}^2 \right\}; \\ &= \frac{1}{3} (3x-1 - \sqrt{-11})(3x-1 + \sqrt{-11}). \end{aligned}$$

(ii). We may test an expression for binomial factors by putting a supposed factor equal to zero, and substituting for one quantity in terms of another.

Thus, to try if $x-1$ be a factor of $x^3 - 7x + 6$, we put $x+1=0$; or, $x=1$. This substitution causes the expression to vanish. $\therefore x-1$ is a factor. Dividing by $x-1$ gives $x^2 + x - 6 = (x-2)(x+3)$.

$$\begin{aligned} (b). \quad x^4 + 1 + \frac{1}{x^4} &= \left(x^4 + 2 + \frac{1}{x^4} \right) - 1 = \left(x^2 + \frac{1}{x^2} \right)^2 - 1 \\ &= \left(x^2 + 1 + \frac{1}{x^2} \right) \left(x^2 - 1 + \frac{1}{x^2} \right) \end{aligned}$$

$$\begin{aligned} \text{similarly, } x^2 + 1 + \frac{1}{x^2} &= \left(x + 1 + \frac{1}{x} \right) \left(x - 1 + \frac{1}{x} \right); \\ \text{and } x^2 - 1 + \frac{1}{x^2} &= x^2 + 2 + \frac{1}{x^2} - 3 = \left(x + \frac{1}{x} \right)^2 - 3 \\ &= \left(x + \sqrt{3} + \frac{1}{x} \right) \left(x - \sqrt{3} + \frac{1}{x} \right). \end{aligned}$$

$$\begin{aligned} \therefore \text{The factor expression is: } & \left(x + 1 + \frac{1}{x} \right) \left(x - 1 + \frac{1}{x} \right) \left(x + \sqrt{3} + \frac{1}{x} \right) \left(x - \sqrt{3} + \frac{1}{x} \right). \end{aligned}$$