

formly distributed over its surface; and also that as it moves from the centre the pressure becomes unequal, till on reaching a point at one-third from the edge the pressure becomes zero at the other edge, as shown in Fig. 1. This is necessarily the limiting position of the resultant for which the whole joint can be in compression; and the theory that the curve of pressure must be within the middle third of the arch ring is therefore equivalent to maintaining that every joint in the arch must be in compression over its entire surface. This corresponds with a much higher degree of stability than many arches actually have; and we may therefore follow the effects produced at the joint as the resultant passes out of the middle third and approaches the intrados. The joint will no longer be entirely in compression, but the pressure will extend over a width equal to three times the distance of the resultant from the inner edge, and the pressure at that edge will be double the mean pressure on that area. As to the remainder of the joint it carries nothing, not being capable of bearing tension. (Fig. 2.) Strictly speaking the outer edge will open but only by an amount that is entirely inappreciable. It is capable of calculation, being proportional to the compression of the stone at the inner edge; but the compressibility of stone is so infinitesimal that practically it cannot be measured; and the "opening" of the joint under such circumstances is therefore entirely theoretical. This position of the resultant is then still compatible with stability. As the resultant continues to approach the edge, the same action continues in an increasing ratio, until the pressure at the inner edge reaches the ultimate strength of the material, when crushing will take place, rotation through a small angle will ensue, and the joint will open visibly at the opposite edge. This is the true limiting position of the resultant in actual cases; and for hard and sound material it may be very near the intrados. That the resultant may be very near the edge of the joint, without causing the failure of the arch, is also held as proved by authors who base their conclusions on experiment. Prof. Cain shows by experiments with wooden models that at the joint of rupture the curve of pressure passed on an average at only one-eighteenth of the width of the joint from the edge without crushing occurring. He also calculates that in the bridge of Neuilly the thrust at the crown might pass within less than two inches from the edge without crushing the material. (7) We must therefore conclude that in the case of actual masonry, the curve of pressure may approach extremely near either to the intrados or the extrados without rupture occurring in the arch. This shows the great difficulty of deducing a theory from existing structures, as so little can be inferred from the bare fact that a structure remains standing.

Let us now take the accompanying outline to represent a portion of the arch-ring in a full arch, either semi-circular or elliptic. We will take it in the usual way to represent by its area the weight of a unit of width in the direction of the axis; and the loading may also be shown as an area representing its average amount per unit of width. We will also consider the loading as placed symmetrically on each side of the key, and a half-arch will therefore be sufficient for the figure. For

the forms of loading we have ordinarily to consider it may be broadly stated that the curve of pressure will resemble a parabola. It is easier to determine its general form than to determine its position in the arch ring; but if such a curve is placed within an arch ring of either circular or elliptic form, it will approach the intrados at opposite points at the haunches, and at those points it will be parallel to the tangent at the intrados. This determines the positions of the joints of rupture, one of which is shown at  $CD$ ; and if we also take a vertical joint  $AB$  at the key, these are the only ones which the curve of pressure will necessarily cut at right angles. The moments of the forces must therefore be taken with reference to the portion  $ABDC$ ; as to divide the arch ring at any other point would introduce a radial force representing friction.

We have then three forces to maintain equilibrium. The thrust  $T$  is horizontal on account of the symmetry. The line  $LW$  is either a vertical through the centre of gravity of the area representing the arch ring itself between the joint of rupture and the key; or it is the resultant of the weight of the arch ring, and the pressures upon its extrados between the same points. When these pressures are all vertical,  $LW$  will also be vertical and may be termed the *Gravity line*. When the joint of rupture is known in position, it becomes known in position, direction and amount; and the resistance  $R$  also becomes known in direction. Thus of the three forces two are known in direction only; but to maintain equilibrium they must intersect in a point at  $L$ , and this may be termed the *Condition of intersection*. The position of the joint of rupture must correspond with this condition.

