

clusion of each exercise, and the strict attention manifested, indicated the deep interest the members took in the instruction." "Miss Lewis's exercises in elocution at the Convention led the teachers to expect special pleasure from her Readings, given on Friday night, in the Town Hall of Stratford, an anticipation which was in no way disappointed, for after her sixth appearance during the evening the crowded audience found themselves in a frame of mind similar to that in which 'Oliver Twist' found himself—they wanted more." Miss Lewis gave her lecture and Readings before the North Grey Association with equal satisfaction and success, and in Toronto and various parts of the Province, her Recitations and Readings have excited the greatest interest. We may add also in connection with this young lady's high qualifications that she is the daughter of Mr. Richard Lewis, the well known elocutionist.

Mathematical Department.

SOLUTIONS TO INTERMEDIATE EXAMINATION PAPERS, JULY, 1881.

ARITHMETIC.

- (a) L. C. M. $= 5 \times 17 \times 47 \times 109 \times 243 = 105,815,565$.
(b) L. C. M. of $4\frac{1}{2}$, 5, $2\frac{1}{2}$, and $3\frac{1}{2} = 3456$ inches, the side of the square.
- (a) Book work.
(b) $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ &c., 5 terms $= \frac{1}{2}$.
(c) $\frac{1}{2} \times \frac{1}{3} =$ product.
 $\therefore \frac{1}{2} \times 5 = 7$ times product.
i.e. $\frac{1}{2} = 7$ times product.
or $\frac{1}{2}$ of $\frac{1}{2} =$ product $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.
- (a) Ans. 173444.
(b) Ans. 5 dys., 21 hrs., 11 min., 53 $\frac{1}{2}$ sec.
- See Hamblin Smith's Arith., Can. Ed., p. 250.
Litre = 1 cub. decimetre $= \frac{1}{1000}$ cub. metre.
1 pt. $= 27\frac{1}{8} = 34.625$ cub. in.
 \therefore 1 litre $= 1.76077 \times 34.625$ cub. in.
1 metre $= 10 \times \frac{1}{1000} \times 34.625 = 39.37$ inches.
- No. dys. $= \frac{6 \times 5 \times 9 \times 3}{7 \times 5 \times 16} = 1\frac{1}{4}$ dys. — Ans.
- No. men $= \frac{3 \times 40 \times 2000 \times 2000 \times 1000}{12 \times 20 \times 1600 \times 1600} = 1562\frac{1}{2}$ men.
- In 15 min. true time, the min. hand will pass over $\frac{1}{10}$ of 15 min. spaces $= 1\frac{1}{2}$ spaces.
In 15 min. true time the hour hand will pass $\frac{1}{4}$ of $\frac{1}{2}$ min. spaces on the face $= 1\frac{1}{8}$ spaces.
Distance apart at time of observation $= 13\frac{1}{2} - 1\frac{1}{8} = 12\frac{3}{8}$ spaces.
- \$3700 yields \$270 int. Rate $= 7\frac{1}{4}\%$. — Ans.
- The company gets 8% compound int. for its money.
 \therefore Sum $(1.08)^2 = 70(1.08) + 70 + 1000$.
Sum $= \frac{1145.60}{1.08 \times 1.08} = \982.17 . — Ans.
See H. Smith's Arith., Can. Ed., p. 343.

ALGEBRA.

- (a) $x^2 + y^2 = (x+y)(x^2 - xy + y^2)$.
(b) $x^3 + y^3 + z^3 - 3xyz = (x^2 + y^2 + z^2 - xy - yz - zx)(x+y+z)$
 $= (x^2 + y^2 + z^2 - xy - yz - zx)(x+y+z)$
 $= (x^2 + y^2 + z^2 - xy - yz - zx)(x+y+z)$
 $= (x^2 + y^2 + z^2 - xy - yz - zx)(x+y+z)$
(1) For $x+z$ write m , for $y-z$ write n and the expression assumes the form
 $m^2 + n^2 - (m+n)(m-n) = (m+n)m$
divide through by $m+n$, using (a)
 $(m^2 - mn + n^2) - (m-n)^2 = mn$, which is an identity on expansion.

(2) Left hand member

$$= (a^2 - bc)^2 - (a^2 - bc)(b^2 - ac)(c^2 - ab) \\ + (b^2 - ac)^2 - (b^2 - ac)(c^2 - ab)(a^2 - bc) \\ + (c^2 - ab)^2 - (c^2 - ab)(a^2 - bc)(b^2 - ac).$$

Factoring each member this is

$$= (a^2 - bc)(a^2 - bc)^2 - (a^2 - bc)(b^2 - ac)(c^2 - ab) \\ + (b^2 - ac)(b^2 - ac)^2 - (b^2 - ac)(c^2 - ab)(a^2 - bc) \\ + (c^2 - ab)(c^2 - ab)^2 - (c^2 - ab)(a^2 - bc)(b^2 - ac).$$

Simplifying the large brackets

$$= a(a^2 - bc)(a^2 + b^2 + c^2 - 3abc) \\ + b(b^2 - ac)(a^2 + b^2 + c^2 - 3abc) \\ + c(c^2 - ab)(a^2 + b^2 + c^2 - 3abc).$$

Multiplying out a , b and c and adding up

$$= (a^3 + b^3 + c^3 - 3abc)(a^2 + b^2 + c^2 - 3abc) \\ = (a^3 + b^3 + c^3 - 3abc)^2.$$

See McLellan's Alg., page 37.

Todhunter's Alg., page 143.

2. The given relation transposed is

$$(a^2 - b^2) + c(a - b) = 0, \text{ or}$$

$$(a - b)(a + b + c) = 0. \text{ Now one at least of the factors must} \\ = 0, \text{ but } a - b \text{ is not } = 0 \text{ since } a \text{ and } b \text{ are unequal.}$$

$$\therefore a + b + c = 0$$

$$\text{i.e. } (a + b + c)(ab + bc + ca) = 0, \text{ which is the required ex-} \\ \text{pression factored.}$$

3. Since the L. C. M. is of only four dimensions, while their product is of six dimensions, the G. O. M. must be of two dimensions. Let it be $x^2 + mx + k$. Divide each of the two given quantities by $x^2 + mx + k$ and put the rems. separately $= 0$, and we get:

$$-k = m(a - m) \text{ and } b = k(a - m);$$

$$\text{also } -m = c - k \text{ and } d = -mk.$$

Now eliminate m and k .

$$k = \frac{b - d}{a} \text{ and } m = \frac{b - d - ac}{a}.$$

$$\text{Hence } -\frac{b - d}{a} = \left(\frac{b - d - ac}{a} \right) \left(a - \frac{b - d - ac}{a} \right).$$

$$\text{i.e. } a(d - b) = (b - d - ac)(a^2 - b + d + ac).$$

$$4. \text{ Sum} = \frac{x^2(x^2 - y^2) + x^2(y^2 - z^2) + y^2(z^2 - x^2)}{(x - y)(y - z)(z - x)}.$$

$$= \frac{(x - y)(y - z)(z - x)(-xy - yz - zx)}{(x - y)(y - z)(z - x)}.$$

$$= -(xy + yz + zx).$$

5. (1) Given expression:

$$= 2 \times \frac{2bc - b^2 - c^2 + a^2}{2bc} \times \frac{2ca - c^2 - a^2 + b^2}{2ca} \times \frac{2ab - a^2 - b^2 + c^2}{2ab}$$

$$= \frac{1}{4a^2b^2c^2} \{a^2 - (b - c)^2\} \{b^2 - (c - a)^2\} \{c^2 - (a - b)^2\},$$

$$= \frac{1}{4a^2b^2c^2} (a + b - c)^2 (a - b + c)^2 (b + c - a)^2, \text{ of which the sq. rt. is}$$

$$\frac{1}{2abc} (a + b - c)(a - b + c)(b + c - a),$$

$$(2) \text{ Sq. rt. } = x^2 + \frac{1}{2}x + \frac{1}{2}, \text{ by inspection.}$$

6. "Every equation of the n th. degree has n roots and only n ." The given expression contains x only to the first degree. Hence if it admits of more than one value for x it must be an identity, not an $= n$. But the expr. vanishes when $x + a = 0$, or $x + b = 0$, or $x + c = 0$. Hence $x = -a, -b, -c$, and the expr. is an identity. See H. Smith's Alg. p. 57.

$$7. \frac{(b+c)(b-c)}{k-a} + \frac{(c+a)(c-a)}{k-b} + \frac{(a+b)(a-b)}{k-c}.$$

Observe that the sum of $(b - c)$, $(c - a)$, $(a - b) = 0$. Thus the expr. would vanish if the remaining part of each fraction disappeared.

$$\text{This would happen if } b + c = k - a, \\ c + a = k - b, \\ a + b = k - c.$$

And we see that these three relations hold when $k = a + b + c$.

8. Let $3x = A$'s income.

$$12y = \text{" expenditure.}$$

$$\therefore 3x - 12y = \text{" saving.}$$

Then from conditions given,

$$2x = B \text{'s income,}$$

$$y = \text{" expenditure.}$$

$$\therefore 2x - y = \text{" saving.}$$

Now their savings are as 4:5,

$$\therefore 5(3x - 12y) = 4(2x - y),$$